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SUPPLY OF TURBULENT ENERGY FOR PARTICLE ACCELERATION IN ASTROPHYSICAL PLASMAS

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ABSTRACT

The equation of transport of the turbulence energy density is analytically solved under simplified assumptions. The simplified solution is coupled to the transport equation of particles accelerated by the turbulence-particle interactions. The result of such a coupling is an analytical solution to the problem of the simultaneous evolution of the turbulence and the accelerated particles in plasmas. This solution applied to the case of the fast magnetosonic turbulence allows us to evaluate the effect of a more the turbulence spectral distribution on the conformation of the particle energy spectrum relative to the case where a constant turbulence energy density is considered during the particle acceleration process. Results indicate that there is an overestimation of the amount of energetic particles when it is assumed a constant supply of turbulence energy density with respect to the more realistic situation derived in this work.

I. INTRODUCTION

Particle acceleration in turbulent plasmas is a common process in nature and has been studied for long time (e.g. Tsytovich 1966, 1970); in spite of the elapsed time, the characterization of the physical mechanisms by means of which the turbulence evolves and transfer energy to the plasma particles is a problem that has not been completely solved (e.g. Miller et al 1997). Within the context of particle acceleration in a turbulent plasma, most of acceleration models assume the existence of a turbulent state such that its energetic content remains constant during the acceleration process; however, the condition for the existence of such state requires of a balance between a number of effects to maintain a constant energy flux, from the energy containers of large scale (i.e. large scale turbulent structures) to the small scale energy containers where occurs the dissipation of the turbulent energy and the corresponding energy transfer to the plasma particles. The study of this two-fold problem leads, for one side, to the establishment of a transport equation of the turbulent energy density W(r,k,t) in the physical space and the wave number space, from the generation region, at large wave lengths, to the spectral regions of dissipation, at short wave lengths, (Section II) , and on the other hand, the establishment of the evolution equation of the number of accelerated particles N(E,t) as a result of the turbulence-particle interaction. Therefore, the equation describing the spectral evolution of the turbulence must be coupled to the equation that determines the energetic evolution of the accelerated particles, however, such a coupling is not a simple one, taking into account that the transport equation of the turbulence is a non-linear differential equation. Nevertheless, (Miller et al. 1996, Miller 1999), derived numerical solutions for the steady state case.

With the aim of obtaining analytical solutions to this problem, in this work we derive simplified solutions to the turbulence transport equation (section III). These solutions determine the instantaneous amount of energy that the turbulence supply. :. This solution must be coupled to the equation of particle energy evolution which analytical solutions were derived by Pérez-Peraza and Gallegos-Cruz, (1994), Gallegos-Cruz and Pérez-Peraza, (1995), which solution determines the instantaneous distribution of the accelerated particles per energy interval, N(E,t) (Section IV). However, those analytical solutions were quantified under the assumption of a constant rate for the turbulence energy density. In order to evaluate particle energy spectra N(E,t), under more realistic situations than those obtained with W(r,k,t) = Cte, in Section V we introduce the simplified analytical solutions obtained in Section III into the analytical solutions of the energy spectra described in Section IV, which implications are discussed in section VI.

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II. TURBULENT ENERGY TRANSPORT

The statistical properties of incompressible MHD turbulence in the tridimensional space of wave numbers (k-space) can be specified as a function of the energy density ε in terms of the wave number and time (e.g. Leith, 1967) as $\varepsilon(t)=\int dk\vec{W}_T(k,t)$ where $\vec{W}_T(k,t)$ represents the tridimensional spectral density of energy fluctuations in units of energy density/volume, in the phase space. The concept of turbulent energy involves the kinetic energy of the fluctuations velocity as well as the magnetic energy associated with the fluctuations of the background magnetic field. The hydrodynamic description of turbulence evolution in terms of an energy diffusion equation in the space-k (Leith, 1967) was introduced in MHD by Zhou and Matthaeus (1990). For the establishment of such equation it is assumed that turbulence evolution occurs through a turbulent cascade, involving several steps, from the large scale energy containers up to the small scale containers where direct dissipation of the turbulence takes place efficiently. In addition of the transport of the energy spectral density of the turbulence in the space of wave numbers, this equation also includes transport in the physical space and terms of supply and dissipation of the turbulent energy:

$$\frac{\partial \ddot{W}_T}{\partial t}+L_x\ddot{W}_T+J_k=S-D \qquad (1)$$

where L_x denotes a linear operator of spatial transport acting on the turbulence energy spectrum $\vec{W}_T$, that may include several effects, such as propagation, convection and expansion; J_k is the transport operator
in the space-k involving changes per time unit of the energy of the fluctuations that have wave numbers close to k. These changes are produced by non linear couplings with the other fluctuations, that is, with those which wave number is $k_l \neq k$. An important hypothesis in this context is to suppose that coupling contributing to J_k only act by rearranging the spectral distribution of the fluctuations, but do not change the amount of energy of the turbulence. Changes in the turbulence energy appear from the energy sources (S) and the energy sinks (D), which k-values are usually out of the dissipation range. The term (S) includes energy supply due to fluctuations of large wave number, and the term (D) includes dissipation effects of the turbulence such as viscous dissipation, thermal conduction, etc. An addition hypothesis is to suppose that the total effect of all these non linear couplings is translated in a local energy spectral transfer in the k-space. On basis to these hypothesis it is assumed the existence of a continuos flux of energy density through the tridimensional space of wave numbers, allowing then to define an energy flux vector ($\ddot{F}$) in the k-space which satisfies

$$J_k=-\vec{\nabla}_k\bullet\ddot{F}(k) \qquad (2)$$

where $\vec{\nabla}_k\bullet\ddot{F}$ indicates the divergence of the vector $\ddot{F}$ in the k-space. According to the energy conservation hypothesis $\left(\int J_k dk=0\right)$ and the assumption that changes in the spectral distribution are produced by non linear couplings, equation (2) may be rewritten (Zhou and Matthaeus, 1990) as

$$\left(\frac{\partial \ddot{W}_T(k)}{\partial t}\right)_{nonlinear}=-\nabla_k\bullet\ddot{F}(k) \qquad (3)$$

where $\ddot{W}_T(k)$ is the tridimensional spectral density of the turbulence energy. For the case of isotropic turbulence it is convenient to define the unidimensional energy spectral density $W_T(k)$, that is the flux vector $\ddot{F}$ through concentric spheres in the tridimensional k-space:

$$F(k)=4\pi k^2\ddot{F}_T(k) \qquad (4)$$

where F(k) is the radial component of the vector $\ddot{F}(k)$ in spherical coordinates, and the other two components are equal to zero by symmetry effects, the tridimensional energy density also satisfies $W_T(k)=4\pi k^2\ddot{W}_T(k)$.
The hypothesis of local energy transport in the k-space and its conservation by non-linear interactions leads to the assume that energy transport is described by the divergence of the tridimensional energy flux, such as established in equation (2). The adoption of a Fick law between the vector flux (with value near the wave number k) and the spectral density $\ddot{W}_T(k)$ leads to diffusion approximation in the k-space within the MHD context:

$$\ddot{F}(k)=-D\nabla_k W_T(k) \qquad (5)$$

A dimensional analysis of the problem (Zhow and Matthaeus, 1990) leads to the establishment of a diffusion equation in the k-space of the following form:

$$\left(\frac{\partial \ddot{W}_T(k)}{\partial t}\right)_{nonlinear} = \frac{\partial}{\partial k}\left[k^2 D \frac{\partial}{\partial k}\left(k^{-2} W_T(k)\right)\right] \qquad (6)$$

This equation describes the diffusion of turbulent energy in the k-space and D is the corresponding diffusion coefficient. According to equation (6) the model of diffusion for non-linear couplings depends only on the energy spectral density, the wave number and the spectral transfer time. Under this basis, equation (1) can be rewritten in the diffusion approximation as

$$\frac{\partial W(k,t)}{\partial t} = \frac{\partial}{\partial k}\left[k^2 D(k) \frac{\partial}{\partial k}\left(k^{-2} W(k,t)\right)\right] - D_{DISS}(k,t) + S(k,t) \qquad (7)$$

III. SIMPLIFIED SOLUTIONS OF THE TRANSPORT OF WAVE NUMBERS

To solve the transport equation (7) it is necessary to know the explicit form of the diffusion coefficient in the space of wave numbers [D(k,t)], the injection [S(k,t)] and the dissipation [D_{diss}(k,t)]. The usual form of the dissipation function is (Leamon et al, 2000):

$$D_{diss}(k, t) = - \gamma (k) E(k) \qquad (8)$$

the equivalent in the hydrodynamic case corresponds to the viscosity, with $\gamma(k) \rightarrow \nu k^2$, where ν is the viscosity coefficient; however, in the MHD context viscosity is not the unique dissipation process acting on the turbulence waves, there is also thermal conduction e interaction of plasma particles with neutral atoms (Braginskii 1962, Eilek, 1979). Regarding the term of injection of turbulence [S(k,t)] it is usually assumed that is injected at a given wave number k_i at a constant temporal rate (e.g. Miller, 1996; Stawicki et al, 2001).
One of the central problems in the solution of equation (7) is the explicit knowledge of the diffusion coefficient [D(k,t)], because it involves the microscopic description of the interaction between the several turbulence excitation states, represented through the wave numbers and characterizing the cascade of energy spectral transfer. Usually it is assumed the following form:

$$D(k) \sim \frac{k^2}{\tau_s(k)} \qquad (9)$$

where $\tau_s(k)$ is the characteristic time for spectral transfer of turbulent energy, depending on the specific approach in consideration. In the Kolmogorov type approach this time for a given wave length is the "eddy turnover time (λ / δv), where δv is the rms of the wave fluctuations, while in the Kraichnan type approach this time is (v_A / δv) times longer:

$$\tau_s(k) = \frac{(2U_B)^{1/2}}{v_A k^{3/4} W_i^{1/2}} \qquad \text{(Kolmogorov)}$$

$$\tau_s(k) = \frac{2U_B}{v_A k^2 W_i} \qquad \text{(Kraichnan)} \qquad (10)$$

where v_A is the Alfven velocity and $U_B = \frac{B_0{}^2}{8\pi}$ is the magnetic energy density of the background magnetic field.

1^{st}. Simplified Solution: Steady State Solution without Dissipation.

This case is the simplest solution of equation (7), and is directly obtained by setting $\frac{\partial W(k,t)}{\partial t} = 0$ and D(k) as given in equation (9) with its corresponding $\tau_S(k)$, so that the obtained solution is

$$W(k) = W_0 k^{-q} \qquad (11)$$

with q = 5/3 for Kolmogorov type phenomenology and q = 3/2 for the Kraichnan type.

2^{nd} Simplified Solution: the case of Negligible Cascade.

This simplified solution is obtained when the wave intensity is enough small, so that the term of cascade can be neglected and equation (7) becomes as follows:

$$\frac{\partial W(k,t)}{\partial t} = - \gamma(k) W(k,t) + S(k,t) \qquad (12)$$

in which case the steady state solution is

$$W(k,t) = \frac{W_0 S(k)}{\gamma(k)}\left[1 - e^{-\gamma(k)t}\right] \qquad (13\ a)$$

whereas the time-dependent solution in the equilibrium approach, that is $(t \rightarrow \infty)$, it is obtained

$$W(k) = \frac{W_0 S(k)}{\gamma(k)} \tag{13 b}$$

In these solutions, the injection function S(k) and dissipation function γ(k) are time constants. For the goal of simplicity it has been assumed (Miller, 1995) that turbulence is injected at one specific wave length at a constant rate between a time t and a time t_f: $S(k,t) = QH(t_f - t)S(k-k_i)$, where Q is the energy deposition rate and H is the step function. When the turbulence begins to act at t=0, then $W = W_0 \delta(k - k_i)$, where W_0 is the initial wave energy density. For the dissipation function there exist several assumptions. In the particular case of fast magnetosonic turbulence in an electron-proton plasma there is the resonant damping (e.g. Ginzburg, 1970) and transit time damping for an isotropic particle distribution (Stix, 1962) in the first case we have:

$$\gamma_R = \sqrt{\frac{\pi B}{16}} v_A |k| \frac{sen^2\theta}{\cos\theta} \left\{ \sqrt{\frac{m_e}{m_p}} + 5e^{-(\beta \cos^2\theta)} \right\} \tag{14}$$

where θ is the wave propagation angle with respect to the background magnetic field. For the second case the rate is (Michalek et al, 1999):

$$\gamma_T = \frac{\pi V_A}{4C} V_A |k| \frac{sen^2\theta}{\cos\theta} \left[1 - \frac{V_A{}^2}{C^2 \cos^2\theta} \right]^2 \frac{U_P}{U_B} \tag{15}$$

where U_p is the energy density of the energetic particles and C the light velocity.

IV. THE TRANSPORT EQUATION OF ENERGETIC PARTICLES.

The coupled solution of the equations of turbulence transport (Eq. 7) and energetic particle transport (e.g. Pérez-Peraza and Gallegos-Cruz, 1994; Gallegos-Cruz and Pérez-Peraza, 1995) in a turbulent plasma is highly difficult to obtain in an analytical way, taking into account that the former is a non-linear equation. However, a first approximation to the coupling problem may be done by incorporating some of the simplified solutions of equation (7) into analytical solutions of the transport equation of energetic particles: assuming that particles are accelerated by fast magnetosonic turbulence in a solar flare scenario Pérez-Peraza and Gallegos-Cruz, (1994) and Gallegos-Cruz and Pérez-Peraza, (1995) have derived the following analytical solutions

$$N(E) = \frac{q_0}{2}(a_f \alpha/3)^{-3/2}(\beta_0^{3/2}\varepsilon_0)^{-1}(\frac{\beta_0}{\beta})^{1/4}(\frac{\varepsilon}{\varepsilon_0})^{1/2}\exp[-(3a_f/\alpha)^{1/2}J_f]$$

(particles / energy unit) (16)

for the steady-state situation, and

$$N(E,t) = \frac{(\beta/\beta_0)^{1/4}(\varepsilon/\varepsilon_0)^{1/2}}{(4\pi\alpha/3)^{1/2}(\beta_0^{3/2})}\left[(\frac{N_0}{t^{1/2}})\exp(-a_f t - \frac{3J_f^2}{4\alpha t}) + (\frac{q_0}{2})(\frac{\pi}{a_f})^{1/2}R_5(\varepsilon_0,\varepsilon)\right]$$

(particles /energy unit x unit time) (17)

in the case of the time-dependent solution.

E is the particle kinetic energy, ε their total energy, α (s^{-1}) the acceleration efficiency and the rest of constants are described in Pérez-Peraza and Gallegos-Cruz, (1994) and Gallegos-Cruz and Pérez-Peraza, (1995).

It is precisely the acceleration efficiency that contains the information about the supply of turbulent energy to the acceleration process (e.g. Miller et al, 1995)

$$\alpha(<k>,t) = \frac{3\pi v_\phi^2}{B_0^2 c}\ln(\frac{\beta c}{v_\phi}) <k> \int_{k_{min}}^{k_{max}} W(k\ r,t)dk$$

(18)

where <k> denotes the wave number average, weighty over the corresponding spectral distribution, that is

$$<k> = (\frac{1}{W_t})\int_{k_{min}}^{k_{max}} kW(k\ r,t)dk \tag{19}$$

where W_t is the total energy density of the turbulence at the time t.

V. RESULTS

For the stationary case we use the obtained wave number spectral distribution, eq. (13b), into equations (18) and (19): for the evaluation of <k> we have considered the interaction between protons and fast magnetosonic waves with wave number k comprised in the range k_{min} - k_{MAX}. For the evaluation of the lower frequency cutoff k_{min} we assume interaction of protons with turbulence which waves length is much greater than the gyroradius of protons $\lambda >> \rho_{protones}$ so that taking $\lambda_{MAX} = 10\rho_{protones}$ and ρ = βε / eB, it is obtained $k_{min} = 3.017 \times 10^{-10} B/\beta\varepsilon$. The upper frequency cutoff is considered to occur at the proton gyrofrequency $\Omega_p = eB / m_p c$, so that $k_{max} = \Omega_{max} / V_A = 4.39128 \times 10^{-8} n^{1/2}$, where V_A is the hydromagnetic Alfven velocity.

For the evaluation of the dissipation function γ we use the combined effect of the coefficients given in eqs. (14) and (15), the later with $U_P/U_B=20$. Once <k> is evaluated, the acceleration efficiency given in eq. (18) is introduced in the stationary energy spectrum, eq. (16). The evaluation for $T=2\times10^6$K, $\tau=0.2$ s, $n=5\times10^9$ cm^{-3}, B= 200 G, $W_0=2$ erg/cm^3 and $E_0=1$ MeV is shown in figure 1 for several angles of wave propagation with respect to the background magnetic field: the solid curve (upper one) shows the distribution of energetic protons per energy interval (energy spectrum) when the employed turbulence spectral distribution is of the Kolmogorov type (eq. 11), that is, constant in the time, whereas the rest of curves show energy spectra with consideration of dissipation effects for the stationary case, that is with eq. (13b).

For the time-dependent case <k> is evaluated with the turbulence spectral distribution given in eq. (13a), with γ evaluated as in the previous case, so that the corresponding acceleration efficiency, eq. (18) is introduced in the non-stationary particle energy spectrum, eq. (17) . Results are shown in Figures 2 and 3, where the solid lines (upper curves) correspond to a spectral distribution of the Kolmogorov type, eq. (11) without dissipation effects, while the rest of curves correspond, as previously mentioned to <k> and α evaluated with eq. (13a). Figure 2 shows the evaluation for different wave propagation angles with respect to the background magnetic field, whereas in Figure 3 the evaluation is done for a fixed angle ($\theta = 10^o$) and different acceleration times.

VI ANÁLYSIS AND CONCLUSIONS

Analysis of figures 1-3 indicates that in both cases, the steady-state and the non-stationary one, there is a notorious diminution of the amount of particles per energy interval, when a more realistic spectral distribution of the turbulence energy density (with dissipation effects) is considered, and this diminution is more notorious as the wave propagation angle increases.

On the other hand, we can appreciate from Figure 3 that there is a slight increase in the amount of energetic particles per energy interval as the acceleration time increases, though for the set of parameters employed here the equilibrium is rapidly reached , around 5 s.

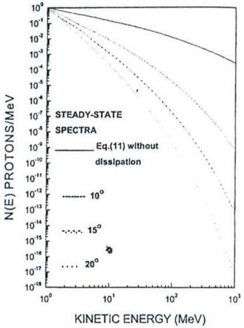

FIGURE 1. Steady-state particle energy spectra for three different wave propagation angles with the following parameters:$T=2\times10^6$ K, $\tau = 0.2s$, $n=5\times10^9$ cm^{-3}, B =100 G. , $W_o=2$ erg/cm^3 y $E_o=1$ MeV.

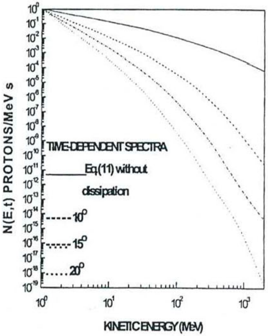

FIGURA 2. Time-dependent particle energy spectra for several wave propagation angles and the following parameters:$T=2\times10^6$ K, $\tau = 0.5s$, $n=5\times10^9$ cm^{-3}, B =100 G. , $W_o=2$ erg/cm^3 , t=5s y $E_o=1$ MeV.

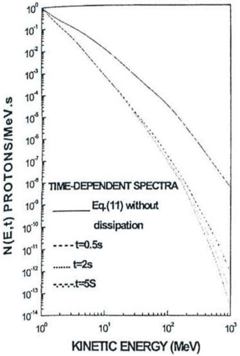

FIGURA 3. Time-dependent particle energy spectra for several values of the acceleration time and a given wave propagation angle, with the following: T=2x10^{6}K, $\tau = 0.5s$, n=5x10^9 cm^{-3} B=100 G. , W_o=2 erg/cm^3 , $\theta = 10^o$ y E_o=1 MeV.

We conclude from this preliminary analytical approach to the problem of solving simultaneously the evolution equations of turbulence and accelerated particles, that the inclusion of dissipation effects of the turbulence during acceleration leads to a decrease of the number of accelerated particles per energy interval, though it is clear this picture may be modified if instead of the simplified case considered here the effect of cascade were included. An analytical study taking into account all these effects will be the matter of a forthcoming work.

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