Parameterization of Proton-Proton Total Cross Sections from 10 GeV to 100 TeV

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Abstract. Present estimations of proton-proton total cross sections at very high energies are obtained from cosmic rays ($> 10^{17}$ eV) by means of some approximations and the knowledge of the measured proton-air cross section at these energies. Besides, total cross sections are measured with present day high energy colliders up to nearly 2 TeV in the center of mass ($\sim 10^{15}$ eV in the laboratory). Here we use a phenomenological model based on the Multiple-Diffraction approach to succesfully describe data at accelerator energies. Then we estimate with it proton-proton total cross sections at cosmic ray energies. On the basis of a forecasting regression analysis we determine confident error bands, analyzing the sensitivity of our predictions to the employed data for extrapolation.

INTRODUCTION

Recently a number of difficulties in uniting accelerator and cosmic ray values of hadronic cross-sections within the frame of the highest up-to-date data have been summarized [1]. Such united picture appears to be highly important for at least, the interpretation of results of new cosmic ray experiments, as the HiRes [2] and in designing proposals that are currently in progress, as the Auger Observatory [3], as well as in designing detectors for future accelerators, as the CERN pp Large Hadron Collider (LHC). Although most of accelerator measurements of $\sigma_{tot}^{\bar{p}p}$ and σ_{tot}^{pp} at center of mass energy $\sqrt{s} \leq 1.8$ TeV are quite consistent among them, this is unfortunately not the case for cosmic ray experiments at $\sqrt{s} > 6$ TeV where some disagreements exist among different experiments. This is also the case among different predictions from the extrapolation of accelerator data up to cosmic ray energies: whereas some works predict smaller values of σ_{tot}^{pp} than those of cosmic ray experiments (e.g. [4,5]) other predictions agree at some specific energies with cosmic ray results (e.g. [6,10]). Dispersion of cosmic ray results are mainly associated to the strong model-dependence of the relation between the basic hadron-hadron cross-section and the hadronic cross-section in air. The later determines the attenuation lenght of hadrons in the atmosphere, which is usually measured in different ways, and depends strongly on the rate (k) of energy dissipation of the primary proton into the electromagnetic shower observed in the experiment: such a cascade is simulated by different Monte Carlo techniques implying additional discrepancies between different experiments. Furthermore, σ_{tot}^{pp} in cosmic ray experiments is determined from σ_{p-air}^{inel} using a nucleon-nucleon scattering amplitude which is frequently in disagreement with most of accelerator data [1]. On the other hand, we dispose of parameterizations (purely theoretically, empirical or semi-empirical based) that fit pretty well the accelerator data. Most of them agree that at the energy of the LHC (14 TeV in the center of mass) or higher (extrapolations) the rise in energy of σ_{tot}^{pp} will continue, though the predicted values differ from model to model. We claim that both the cosmic ray and parameterization approaches must complement each other in order to draw the best description of the hadronic cross-section behavior at ultra high energies. However, the present status is that due to the fact that interpolation of accelerator data is nicely obtained with most of parameterization models, it is expected that their extrapolation to higher energies be highly confident: as a matter of fact, parameterizations are usually based in a short number of fundamental parameters, in contrast with the difficulties found in deriving σ_{tot}^{pp} from cosmic ray results [1]. If extrapolation from parameterization models is correct this would imply that σ_{p-air}^{inel} should be smaller, which would have important consequences for development of high energy cascades.

With the aim of contributing to the understanding of this problem, in this paper we first briefly analyze in the first two sections the way estimations are done for proton-proton total cross sections from accelerators as well as from cosmic rays.

We find serious discrepancies among both estimations. In the third section, on the basis of the Multiple Diffraction model applied to accelerator data, we predict pp total cross section values with smaller errors than with the standard techniques. We conclude with a discussion about the quality of present cosmic ray estimations.

I HADRONIC σ_{tot}^{pp} FROM ACCELERATORS

Since the first results of the Intersecting Storage Rings(ISR) at CERN arrived in the 70s, it is a well established fact that σ_{tot}^{pp} rises with energy ([11,12]). The CERN $S\bar{p}pS$ Collider found this rising valid for $\sigma_{tot}^{\bar{p}p}$ as well [14]. Later, the Tevatron confirmed that for $\sigma_{tot}^{\bar{p}p}$ the rising still continues at 1.8 TeV, even if there is a disagreement among the diferent experiment values as for the exact value ([15,16]). A thoroughful discussion on these problems may be found in [17,18]. It remains now to estimate the amount of rising of the total cross section at those energies. Let us resume the standard technique used by accelerator experimentalists [5].

Using a semi-empirical parameterization based on Regge theory and asymptotic theorems experimentalists have succesively described their data from the ISR to the $S\bar{p}pS$ energies. It takes into account all the available data for σ_{tot}^{pp}, ρ^{pp}, $\sigma_{tot}^{\bar{p}p}$ and $\rho^{\bar{p}p}$, where $\rho^{pp,\bar{p}p}$, is the real part of the $(pp,\bar{p}p)$ forward elastic amplitude at $t=0$. The fits are performed using the once-subtracted dispersion relations:

$$\rho_\pm(E)\sigma_\pm(E) = \frac{C_s}{p} + \frac{E}{\pi p}\int_m^\infty dE'p'\left[\frac{\sigma_\pm(E')}{E'(E'-E)} - \frac{\sigma_\mp(E')}{E'(E'+E)}\right] \tag{1}$$

where C_s is the substraction constant. The expression for $\sigma_{tot}^{pp,\bar{p}p}$ is:

$$\sigma_{-,+}^{tot} = A_1E^{-N_1} \pm A_2E^{-N_2} + C_0 + C_2[ln(s/s_0)]^\gamma \tag{2}$$

where - (+) stands for pp ($\bar{p}p$) scattering. Cross sections are measured in mb and energy in GeV, E being the energy measured in the lab frame. The scale factor s_0 have been arbitrarily chosen equal to 1 GeV^2. The most interesting piece is the one controling the high-energy behaviour, given by a $ln^2(s)$ term, in order to be compatible, asymptotically, with the Froissart-Martin bound [19]. The parameterization assumes σ_{tot}^{pp} and $\sigma_{tot}^{\bar{p}p}$ to be the same asymptotically. This is justified from the very precise measurement of the ρ parameter at 546 GeV [28].

The eight free parameters are determined by a fit which minimizes the χ^2 function

$$\chi^2 = \chi^2_{\sigma_{\bar{p}p}} + \chi^2_{\rho_{\bar{p}p}} + +\chi^2_{\sigma_{pp}} + \chi^2_{\rho_{pp}} \tag{3}$$

The fit has proved its validity predicting from the ISR data (ranging from 23 to 63 GeV in the center of mass), the $\sigma_{tot}^{\bar{p}p}$ value later found at the $S\bar{p}pS$ Collider, one order of magnitude higher in energy (546 GeV) [13,14]. With the same well-known technique and using the most recent results it is possible to get estimations for σ_{tot}^{pp} at the LHC and higher energies. These estimations, together with our present

TABLE 1. $\sigma_{tot}^{\bar{p}p}$ data from high energy accelerators: fits values are from [5].

$\sqrt{s}$ (TeV)		σ_{tot}^{pp} (mb)
0.55	Fit	61.8 ± 0.7
	UA4	62.2 ± 1.5
	CDF	61.5 ± 1.0
1.8	Fit	76.5 ± 2.3
	E710	72.8 ± 3.1
	CDF	80.6 ± 2.3
14	Fit	109.0 ± 8.0
30	Fit	126.0 ± 11.0
40	Fit	130.0 ± 13.0

experimental knowledge for both σ_{tot}^{pp} and $\sigma_{tot}^{\bar{p}p}$ are plotted in figure 1. We have also plotted the cosmic ray experimental data from AKENO (now AGASSA) [23] and the Fly's Eye experiment [24,25]. The curve is the result of the fit described in [5]. The increase in σ_{tot}^{pp} as the energy increases is clearly seen. Numerical predictions from this analysis are given in Table 1. It should be remarked that at the LHC energies and beyond the fitting results display relatively high error values, equal or bigger than 8 mb. We conclude that it is necessary to look for ways to reduce the errors and make the extrapolations more precise.
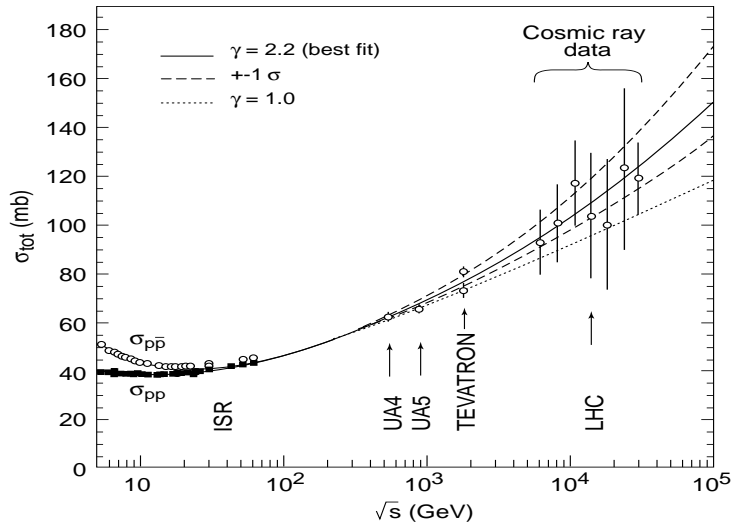

FIGURE 1. Experimental σ_{tot}^{pp} and $\sigma_{tot}^{\bar{p}p}$ with the prediction of [5].

II HADRONIC σ_{tot}^{pp} FROM COSMIC RAYS

Cosmic rays experiments give us σ_{tot}^{pp} in an indirect way: we have to derive it from cosmic ray extensive air shower (EAS) data. But, as summarized in [1] and widely discussed in the literature, the determination of σ_{tot}^{pp} is a rather complicated process with at least two well differentiated steps. In the first place, the primary interaction involved in EAS is proton-air; what it is determined through EAS is the p-inelastic cross section, σ_{inel}^{p-air}, through some measure of the attenuation of the rate of showers, Λ_m, deep in the atmosphere:

$$\Lambda_m = k\lambda_{p-air} = k \ \frac{14.5m_p}{\sigma_{inel}^{p-air}} \tag{4}$$

The k factor parameterizes the rate at which the energy of the primary proton is dissipated into electromagnetic energy. A simulation with a full representation of the hadronic interactions in the cascade is needed to calculate it. This is done by means of Monte Carlo techniques [20–22]. Secondly, the connection between σ_{inel}^{p-air} and σ_{tot}^{pp} is model dependent. A theory for nuclei interactions must be used. Usually is Glauber's theory [7,8]. The whole procedure makes hard to get a general agreed value for σ_{tot}^{pp}. Depending on the particular assumptions made the values may oscillate by large amounts, from as low to 133 ± 10 mb [23] to nearly 165 ± 5 mb [26] and even 175_{-27}^{+40} [27] at $\sqrt{s} = 40$ TeV.

¿From this analysis the conclusion is that cosmic-ray estimations of σ_{tot}^{pp} are not of much help to constrain extrapolations from accelerator energies [1]. Conversely we could ask if those extrapolations could not be used to constrain cosmic-rays estimations.

III A MULTIPLE-DIFFRACTION APPROACH FOR σ_{tot}^{pp}

Let us tackle the mismatching between accelerator and cosmic ray estimations using the multiple-diffraction model [9]. The elastic hadronic scattering amplitude for the collision of two hadrons A and B is described as

$$F(q,s) = i \int_0^\infty b \ db \left[1 - e^{i\Xi(b,s)} \right] J_o(qb) \tag{5}$$

where $\Xi(b,s)$ is the eikonal, b the impact parameter, J_o the zero-order Bessel function and $q^2 = -t$ the four-momentum transfer squared. The eikonal can be expressed at first order as $\Xi(b,s) = \langle G_A G_B f \rangle$, where G_A and G_B are the hadronic form factors, f the averaged elementary amplitude among the constituent partons and the brackets denote the symmetrical two-dimensional Fourier transform. Given the elastic amplitude $F(q,s)$, σ_{tot}^{pp} may be evaluated with the help of the optical theorem:

$$\sigma_{tot}^{pp} = 4 \ \pi \ ImF(q=0,s) \tag{6}$$

Multiple-diffraction models differ one from another by the particular choice of parameterizations made for G_A and G_B and the elementary amplitude f. In the case of identical particles, as is our case, $G_A = G_B = G$. For our purposes we follow the parameterization developed in [10] which has the advantage of using a small set of free parameters, five in total: two of them (α^2, β^2) associated with the form factor G

$$G = (1 + \frac{q^2}{\alpha^2})^{-1}(1 + \frac{q^2}{\beta^2})^{-1} \tag{7}$$

and three energy-dependent parameters (C, a, λ) associated with the elementary complex amplitude f

$$f(q, s) = Ref(q, s) + iImf(q, s) \tag{8}$$

$$Imf(q, s) = C\frac{1 - \frac{q^2}{a^2}}{1 - \frac{q^4}{a^4}} \; ; Ref(q, s) = \lambda(s)Imf(q, s) \tag{9}$$

We get

$$ImF(q = 0, s) = \int_0^\infty \left[1 - e^{-\Omega(b,s)} \cos\left\{\lambda\Omega(b, s)\right\}\right] b \; db \; J_o(q, b) \tag{10}$$

with the opacity $\Omega(b, s)$ given as:

$$\Omega(b, s) = \int_o^\infty G^2 \; Imf(q, s) \; J_o(q, b) \; qdq \tag{11}$$

$$\Omega(b, s) = C\{E_1 K_0(\alpha b) + E_2 K_0(\beta b) + E_3 K_{ei}(ab) + \\ E_4 K_{er}(ab) + b\left[E_5 K_1(\alpha b) + E_6 K_1(\beta b)\right] \} \tag{12}$$

where k_0, k_1, k_{ei}, and k_{er} are the modified Bessel functions, and E_1 to E_6 are functions of the free parameters. The proton-proton total cross-section is directly determined by the expression

$$\sigma_{tot}^{pp} = 4\pi \int_o^\infty b \; db \left\{1 - e^{-\Omega(b,s)} \cos\left[\lambda\Omega(b, s)\right]\right\} \; J_o(q, b) \tag{13}$$

This equation was numerically solved. The overall procedure is done in a three step process.

- First, we determine the parameters of the model by fitting all the pp as well as $\bar{p}p$ accelerator data (differential elastic cross sections and ρ values). in the interval $13.8 \leq \sqrt{s} \leq 1800$ GeV. The obtained values are listed in Table 2.

- Secondly, and most important, we estimate an error band for each of the energy-dependent parameters. To this end we introduce the so-called forecasting technique, based on the multiple linear regression theory. It consists in determining a prediction equation por each free parameter. This allows to set a confidence band for each parameter and the confidence band for the predicted total cross section. The technique is explained in detail elsewhere [29].

- Finally, we proceed to extrapolate our results to high energies. Results are summarized in Table 3a and plotted in figure 2b, together with cosmic ray data. As a comparison, we list in Table 3b the extrapolated values obtained when only pp data, covering a much smaller t-range interval ($13.8 \leq \sqrt{s} \leq 62.5$ GeV), was used. This was the method in [10], but their extrapolated values were given without quoting any errors.

It may be argued that σ_{tot}^{pp} and $\sigma_{tot}^{\bar{p}p}$ are different at high energies: This is the "Odderon hypothesis", which as indicated in Section I, has been very much weakened [28]. Taking this into account, in our multiple-diffraction analysis it is assumed the same behaviour for σ_{tot}^{pp} and $\sigma_{tot}^{\bar{p}p}$ at high energy.

Of course, if we limit our fitting calculations to the accelerator domain $\sqrt{s} \leq 62.5$ GeV (Table 3b), our results are the same as those obtained in [10]. In that case, the σ_{tot}^{pp} values obtained when extrapolated to ultra high energies seem to confirm the highest quoted values of the cosmic ray experiments [26,27]. That would imply the extrapolation cherished by experimentalists is wrong. But the prediction $\sigma_{tot}^{pp} = 91.6$ mb at the Fermilab Collider energy (1.8 TeV) is too high, first, and secondly, difficult to interpret, as no error is quoted in that work. In Table 1 we see that the measured $\sigma_{tot}^{\bar{p}p}$ at 546 GeV is smaller than the predicted σ_{tot}^{pp} by near 8 mb, and in the case of 1.8 TeV by more than 15 mb, which no available model is able to explain [18]. Also it can be noted that the extrapolation from figure 2a to ultra high energies may claim agreement with the analysis carried out in [26] and the experimental data of the Fly's Eye [27], and even with the Akeno collaboration [23], because its errors are so big that overlap with the errors reported in [27]. That is, such an extrapolation, Fig.2a, produces an error band so large at cosmic ray energies that any cosmic ray results become compatible with results at accelerator energies, as it is claimed in [10]. However, when additional data at higher accelerator energies are included (Table 3b), both the predicted values and the error band obviously change. This can be clearly seen in figure 2b, where we have considered data at 0.546 TeV and 1.8 TeV (see Table 1) in which case the predicted value of σ_{tot}^{pp} from our extrapolation at $\sqrt{s} = 40$ TeV, $\sigma_{tot}^{pp} = 131.7_{-4.6}^{+4.8}$ mb is incompatible with those in [26,27] by several standard deviations, though no so different to the Akeno results and the predicted value in [5].

Concerning the quoted error bands, the forecasting technique has reduced the errors, as is seen in figure 2(b), nearly by a factor of 3, as compared with the results quoted in Table 1.

TABLE 2. Values of the parameters C, α^{-2}, λ at each energy. They are obtained from fitting the pp ($\bar{p}p$) differential cross sections and ρ^{pp} ($\rho^{\bar{p}p}$) data in the interval $13 \leq \sqrt{s} \leq 62.5$ ($546 \leq \sqrt{s} \leq 1800$ GeV)

$\sqrt{s}$ (TeV)	C (GeV^{-2})	α^{-2} (GeV^{-2})	λ
13.8	9.97	2.092	-0.094
19.4	10.05	2.128	0.024
23.5	10.25	2.174	0.025
30.7	10.37	2.222	0.053
44.7	10.89	2.299	0.079
52.8	11.15	2.370	0.099
62.5	11.50	2.439	0.121
546	???	???	???
630	???	???	???
1800	???	???	???

IV   CONCLUSIONS:

It has been shown in this work that highly confident predictions of high energy σ_{tot}^{pp} values are strongly dependent on the energy range covered by experimental data and the available number of those data values. In particularly, we show that if we limit our study of determining σ_{tot}^{pp} at cosmic ray energies from extrapolation of accelerator data of $\sqrt{s} \leq 62.5$ GeV, then results are compatible with most of cosmic ray experiments and other prediction models, because the predicted error band is so wide that covers their corresponding error bands (Fig. 2a). However, as x the included data in our calculations extends to higher energies, that is, when all experimental available data is taken into account, the estimated values for σ_{tot}^{pp} obtained from extrapolation and those obtained from cosmic ray experiments are only compatible, within the error bars, with the Akeno results (Fig. 2b). It should be noted that our predictions are compatibles with other prediction studies [5]. Taken all these convergences at face value, as indicating the most probable σ_{tot}^{pp} value, we conclude that if predictions from accelerator data are correct, hence, it should be of great help to normalize the corresponding values from cosmic ray experiments, as for instance by keeping the (k) parameter as a free one, as it is done for instance in [31]. The k value found will greatly help the tunning of the complicated Monte Carlo calculations used to evaluate the development of the showers induced by cosmic rays in the upper atmosphere. In summary, extrapolations from accelerator

TABLE 3. Predicted σ_{tot}^{pp} from fitting accelerator data: (a) extrapolation including data at 546 GeV and 1.8 TeV (the two first values are interpolations). (b) extrapolation with data at $\sqrt{s} \leq 62.5$ GeV; Experimental values are displayed in Table 1

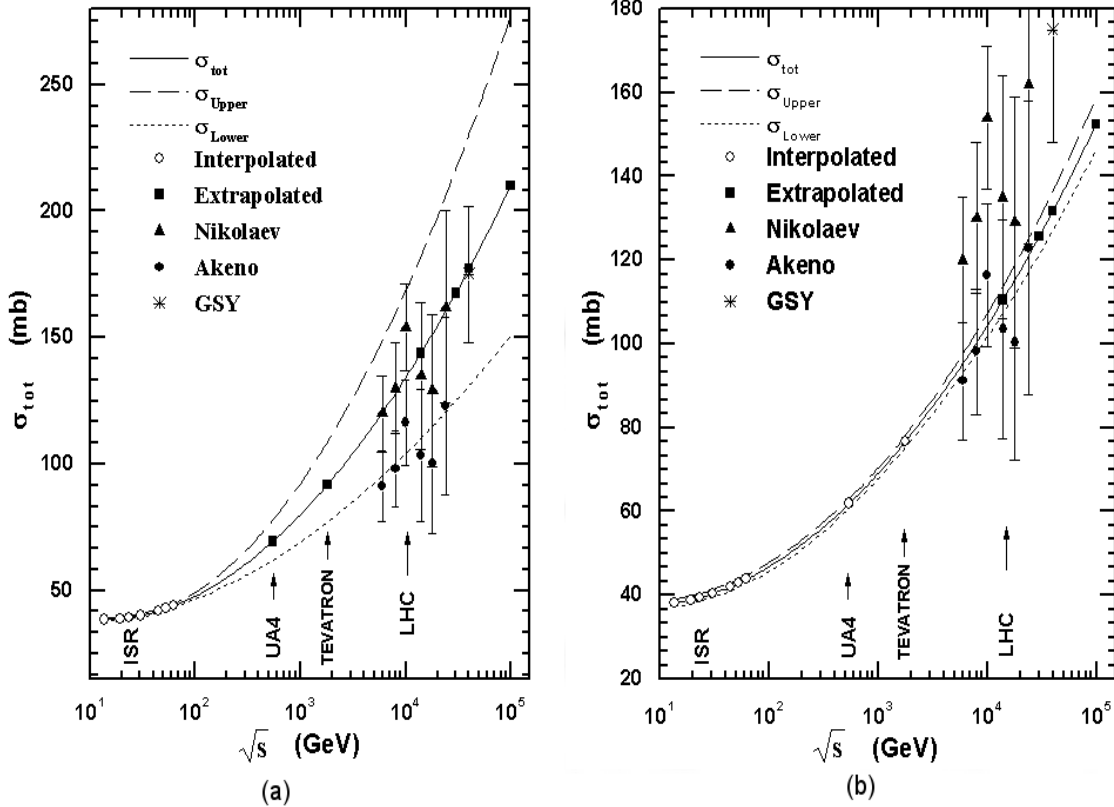| $\sqrt{s}$ (TeV)	σ_{tot}^{pp} (mb)		σ_{tot}^{pp} (mb)	
0.55	Intrp.	$61.91^{+1.2}_{-1.1}$	Extrp.	$69.39^{+8.4}_{-7.4}$
1.8	Intrp.	76.78 ± 1.4	Extrp.	$91.74^{+16.9}_{-14.7}$
14	Extrp.	$110.49^{+3.2}_{-3.1}$	Extrp.	$143.86^{+38.6}_{-33.5}$
30	Extrp.	$125.63^{+4.3}_{-4.1}$	Extrp.	$167.64^{+48.9}_{-42.6}$
40	Extrp.	$131.71^{+4.8}_{-4.6}$	Extrp.	$177.23^{+53.1}_{-46.3}$
100	Extrp.	$152.45^{+6.4}_{-6.2}$	Extrp.	$210.06^{+67.6}_{-59.1}$
	(a)		(b)	

FIGURE 2. Predictions (black squares) of σ_{tot}^{pp} (a) width data at $\sqrt{s} \leq 62.5$ GeV; (b) including data at 546 GeV and 1.8 TeV. Open circles denote the interpolations. Notice the different vertical scales.

data should be used to constraint cosmic ray estimations.

REFERENCES

1. Engel R., Gaisser T.K., Lipari P., Stanev T., *Phys. Rev. D* **58**, 014019 (1998).
2. See http://sunshine.chpc.utah.edu/research/cosmic/hires/
3. The Pierre Auger Project Design Report. *Fermilab report* (Feb. 1997).
4. Donnachie, A and Landshoff, P.V. *Phys. Lett. B* **296**, 227 (1992).
5. Augier, C. et al *Phys. Lett. B* **315**, 503 (1993a).
6. M.M.Block, F. Halzen and T. Stanev, Phys. Rev. Lett. **83**, 4926 (1999).
7. Glauber, R.J., 1956, *Lectures in Theoretical Physics* (Reading: Interscience, N.Y.).
8. Glauber R.J., Matthiae G., *Nucl. Phys. B* **21**, 135 (1970).
9. Glauber, R.J., Velasco, J. *Phys. Lett. B* **147**, 380 (1984).
10. Martini, A.F. and Menon, M.J. *Phys. Rev. D* **56**, 4338 (1997).
11. Amaldi, U. et al *Phys. Lett. B* **44**, 11 (1973).
12. Amendolia, S.R. et al *Phys. Lett. B* **44**, 119 (1973).
13. Amaldi, U. et al. *Phys. Lett. B* **66**, 390 (1977).
14. Bozzo, M. et al *Phys. Lett. B* **147**, 392 (1984).
15. Amos, N. et al, *Phys. Rev. Lett.* **63**, 2784 (1989).
16. Abe et al. *Phys. Rev. D* **50**, 5550 (1994).
17. Matthiae, G. *Rep. Prog. Phys.* **57**, 743 (1994).
18. *Proc. IXth Blois Workshop*, **1999**, on Elastic and Diffractive Scattering, Protvino, Russia.
19. Froissart, M. *Phys. Rev.* **123**, 1053 (1961); Martin, A. 1966, Nuovo Cimento 42, 930 (1966).
20. C. L. Pryke, e-Print Archive: astro-ph/0003442
21. A. M. Hillas, Nucl. Phys. B (Proc. Supp.) **52B**, 29 (1997).
22. R. S. Fletcher et al., Phys. Rev. D **50**, 5710 (1994).
23. Honda, M. et al *Phys. Rev. Lett.* **70**, 525 (1993).
24. Baltrusatis, R.M. et al *Phys. Rev. Lett.* **52**, 1380 (1984).
25. Baltrusatis, R.M. et al *Porc. 19Th ICRC*, La Jolla (1985).
26. Nikolaev, N.N. *Phys. Rev. D* **48**, R1904 (1993).
27. Gaisser, T.K., Sukhatme, U.P., Yodh, G.B. *Phys. Rev. D* **36**, 1350 (1987).
28. Augier, C. et al *Phys. Lett. B* **316**, 448 (1993b).
29. Perez-Peraza, J., Gallegos-Cruz, A., Velasco, J., Sanchez-Hertz, A. Faus-Golfe, A. and Alvarez-Madrigal, M. *"Prediction of p-p total cross-sections with highly confident uncertainty band", in AIP Proceedingg of the Metepec, Puebla*, International Workshop on Observing ultra high energy cosmic rays from space and earth, august (2000).
30. Velasco, J., Perez-Peraza, J., Gallegos-Cruz, A., Alvarez-Madrigal, M., Faus-Golfe, A. and Sanchez-Hertz, A. *Proc. 26 th ICRC, UTAH*, **1**, 198 (1999).
31. M.M.Block, F. Halzen and T. Stanev, Phys. Rev. D **62**, 077501 (2000).