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Parametrization of Hadronic Cross Sections in the Range 10^{-2}-10^{2} TeV

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Abstract. Present estimations of proton-proton total cross sections at very high energies are obtained from cosmic rays ($> 10^{17}$ eV): by means of some approximations, it is possible to get a value for the proton-proton total cross section from the knowledge of the proton-air cross section at these energies. Besides, total cross sections are measured with present day high energy colliders up to nearly 2 TeV in the center of mass ($\sim 10^{15}$ eV in the lab.): several theoretical, empirical or semi-empirical parametrizations, very successful for interpolation at accelerator energies, can then be used to extrapolate the measured value to cosmic ray energies and get a reasonable estimation of cross sections at higher energies ($\sim 10^{17}$ eV). Here we use a phenomenological model based on the Multiple-Diffraction approach to estimate proton-proton total cross sections at cosmic ray energies: on the basis of a forecasting regression analysis we determine confident error bands. We show that our predictions are highly sensitive to the employed data for extrapolation. When both cross section estimations - from accelerators data and most cosmic rays results - are compared, a disagreement is observed, amounting to more than 10%, showing a discrepancy beyond statistical errors.

INTRODUCTION

Recently [1]it has been sumarized a number of difficulties in uniting accelerator and cosmic ray values of hadronic cross-sections within the frame of up-to-date data. Such united picture appears to be highly important for at least, the interpretation of results of new cosmic ray experiments, as the HiRes [2] and in designing proposals that are currently in progress, as the Auger Observatory [3], as well as in designing detectors for future accelerators, as the CERN pp Large Hadron Collider (LHC). Although most of accelerator measurements of σ_{tot}^{pp} at center of mass energy $\sqrt{s} \leq 1.8$ TeV are quite consistent among them, this is unfortunately not the case for cosmic ray experiments at $\sqrt{s} > 6$ TeV where some disagreements exist among different experiments. This is also the case among different predictions from the

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extrapolation of accelerator data up to cosmic ray energies: whereas some works predict smaller values of σ_{tot}^{pp} than those of cosmic ray experiments (e.g. [4,5]) other predictions agree at some specific energies with cosmic ray results (e.g. [6,7]). Dispersion of cosmic ray results are mainly associated to the strong model-dependence of the relation between the basic hadron-hadron cross-section and the hadronic cross-section in air. The later determines the attenuation lenght of hadrons in the atmosphere, which is usually measured in different ways, and depends strongly on the rate ($\overline{k}$) of energy dissipation of the primary proton into the electromagnetic shower observed in the experiment: such a cascade is simulated by different Monte Carlo techniques implying additional discrepancies between different experiments. Furthermore, σ_{tot}^{pp} in cosmic ray experiments is determined from σ_{p-air}^{inel} using a nucleon-nucleon scattering amplitud which is frequently in disagreement with most of accelerator data [1]. On the other hand, parametrizations (purely theoretically, empirical or semi-empirical based) fit pretty well the accelerator data, and most of them agree that at the energy of the LHC (14 TeV in the center of mass) or higher (extrapolations) the rise in energy of σ_{tot}^{pp} will continue, though the predicted values differs from model to model. We claim that both the cosmic ray and parametrization approaches must complement each other in order to draw the best description of the hadronic cross-section behavior at ultra high energies. However, the present status is that due to the fact that interpolation of accelerator data is nicely obtained with most of parametrization models, it is expected that their extrapolation to higher energies be highly confident: as a matter of fact, parametrizations are usually based in a short number of fundamental parameters, in contrast with the difficulties found in deriving σ_{tot}^{pp} from cosmic ray results [1]. If extrapolation from parametrization models is correct this would imply that σ_{p-air}^{inel} should be smaller, which would have important consequences for development of high energy cascades. With the aim of contribute in the field of parametrization techniques, we present here our results of extrapolation made on basis of the Multiple Diffraction model [8–10] according to the specific version developed in [6].

I HADRONIC σ_{tot}^{pp} FROM ACCELERATORS

Since the first results of the Intersecting Storage Rings(ISR)at CERN arrived in the 70s, it is a well established fact that σ_{tot}^{pp} rises with energy ([11,12]). The CERN $Sp\bar{p}S$ Collider found this rising valid for $\sigma_{tot}^{\bar{p}p}$ as well [13]. Later, the Tevatron confirmed that for $\sigma_{tot}^{\bar{p}p}$ the rising still continues at 1.8 TeV, evenif there is a disagreement among the diferent experiment values as for the exact value ([14,15]). A thoroughful discussion on these problems may be found in [16,17]. The agreement reached upon this point is that it remains now to estimate the amount of rising of the total cross section at those energies. In this contribution we study two different approaches to the problem. Let us start first with a standard technique used by accelerator experimentalists and then in next section we will briefly describe the technique used by cosmic ray experimentalists. Works based on accelerator data use the available data for σ_{tot}^{pp} and ρ, the real part of the forward elastic amplitude at $t = 0$ [5]. The fits are performed using the once-subtracted dispersion relations:

$$\rho_{\pm}(E)\sigma_{\pm}(E) = \frac{C_s}{p} + \frac{E}{\pi p}\int_m^{\infty} dE' p' \left[\frac{\sigma_{\pm}(E')}{E'(E'-E)} - \frac{\sigma_{\mp}(E')}{E'(E'+E)}\right] \quad (1)$$

where C_s is the subtraction constant. The expression for σ_{tot}^{pp} is: $\sigma_{tot}^{\bar{p}p} = A_1 E^{-N_1} \pm A_2 E^{-N_2} + C_0 + C_2[ln(s/s_0)]^{\gamma}$ where + (-) stands for pp ($\bar{p}p$) diffusion. σ_{tot}^{pp} is measured in mb and energy in GeV, E being the energy measured in the lab frame. The scale factor s_0 have been arbitrarily chosen equal to 1 GeV^2. The most interesting piece is the one controling the high-energy behaviour, given by a $ln^2(s)$ term, in order to be compatible, asymptotically, with the Froissart-Martin bound [18]. The parametrization assumes σ_{tot}^{pp} and $\sigma_{tot}^{\bar{p}p}$ to be the same asymptotically. The model is described by a set of eight parameters, whose best fits are determined by a χ^2 minimization procedure. It is a *simultaneous* fit of σ_{tot}^{pp} and ρ which minimizes the χ^2 function $\chi^2 = \chi^2_{\sigma_{\bar{p}p}} + \chi^2_{\rho_{\bar{p}p}} + +\chi^2_{\sigma_{pp}} + \chi^2_{\rho_{pp}}$. The fit has proved its validity predicting, from the ISR data (23-63 GeV in the center of mass), the $\sigma_{tot}^{\bar{p}p}$ value found at the $S\bar{p}pS$ Collider (546 GeV), one order of magnitude higher in energy [19,13]. With the same well-known technique and using the most recent results it is possible to get estimations for σ_{tot}^{pp} at the energies of the LHC and beyond [5]. These estimations, together with our present experimental knowledge for both σ_{tot}^{pp} and $\sigma_{tot}^{\bar{p}p}$ are summarized in Table 1 and plotted in figure 1. We have also plotted the cosmic ray experimental data from AKENO (now AGASSA) [20] and the Fly's Eye experiment [21,22]. The curve is the result of the fit described in [5]. The increase in σ_{tot}^{pp} as the energy increases is clearly seen. The main conclusion from this analysis based on accelerators results are the predictions ~~in~~ shown in Table 1. It should be remarked that the previuos fitting results display relatively high error values quoted from the χ^2 minimization procedures, and that the model itself depends on a relatively high number of free parameters. Our conclusion from these remarks is the need to optimize the Multiple Scatering model in order to reduce the prediction errors and to render easier the computing tecnique.

II HADRONIC σ_{tot}^{pp} FROM COSMIC RAYS

Cosmic rays experiments give us σ_{tot}^{pp} as derived from cosmic ray extensive air shower (EAS)data. But, as sumarized in [1] and widely discussed in the literature, the determination of (σ_{tot}^{pp}) is a rather complicated process with at least two well differentiated steps: First the primary interaction involved in EAS is proton-air; what it is determined through EAS is the p-inelastic cross section, σ_{inel}^{p-air}, through some measure of the attenuation of the rate of showers, Λ_m, deep in the atmosphere:

TABLE 1. $\sigma_{tot}^{\bar{p}p}$ data from high energy accelerators: fits values are from [5].

$\sqrt{s}$ (TeV)		$\sigma_{tot}^{\bar{p}p}$ (mb)
0.55	Fit	61.8 ± 0.7
	UA4	62.2 ± 1.5
	CDF	61.5 ± 1.0
1.8	Fit	76.5 ± 2.3
	E710	72.8 ± 3.1
	CDF	80.6 ± 2.3
14	Fit	109.0 ± 8.0
30	Fit	126.0 ± 11.0
40	Fit	130.0 ± 13.0

$\Lambda_m = k\lambda_{p-air} = k\left(14.5 m_p/\sigma_{inel}^{p-air}\right)$. The k factor parameterizes the rate at which the energy of the primary proton is dissipated into electromagnetic energy. A simulation with a full representation of the hadronic interactions in the cascade is needed to calculate it. This is done by means of Monte Carlo techniques. Secondly, the connection between σ_{inel}^{p-air} and σ_{tot}^{pp} is model dependent. A theory for nuclei interactions must be used. Usually is Glauber's theory [8,10]. This procedure makes hard to get a general agreed value for σ_{tot}^{pp}. Depending on the particular assumptions made the values may oscillate by large amounts, from as low to 133 ± 10 mb [20] to nearly 165 ± 5 mb [23] and even 175_{27}^{40} [24] at $\sqrt{s} = 40$ TeV. From the previous analysis the conclusion is that cosmic-ray estimations of σ_{tot}^{pp} are not of much help to constrain extrapolations from accelerator energies [1]. Conversely we could ask if those extrapolations could not be used to constrain cosmic-rays estimations.

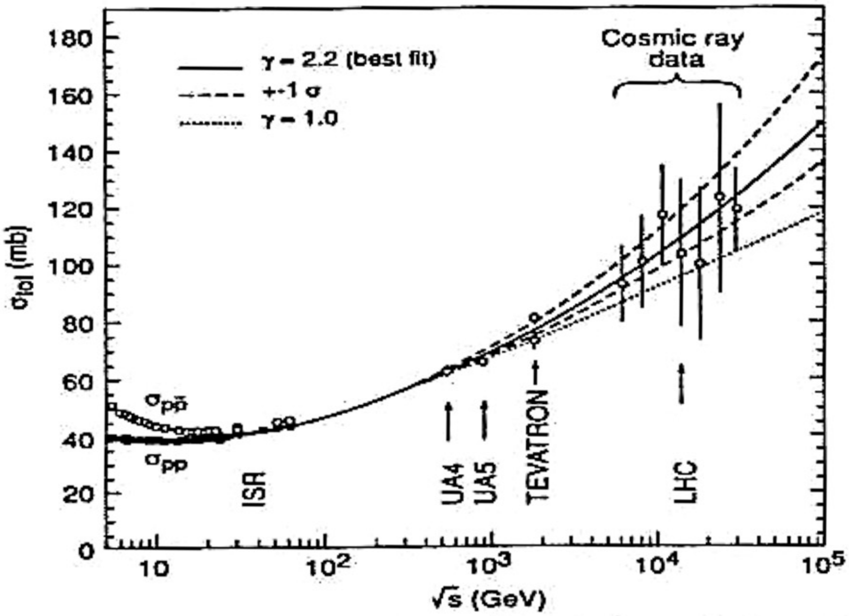

FIGURE 1. Experimental σ_{tot}^{pp} and $\sigma_{tot}^{\bar{p}p}$ with the prediction of [5].

III A PHENOMENOLOGICAL MULTIPLE-DIFFRACTION APPROACH FOR σ_{tot}^{pp}

From the conclusions of the two previous sections we have proceeded to tackled the mentioned problems using the multiple-diffraction model [8,9]. In a recent phenomenological version of it, within the frame of the eikonal formalism, a new approach has been developed under the consideration of a complex elementary amplitud (parton-parton) [6] and azimuthal simmetry in the collision of two hadrons A and B. The elastic hadronic scattering amplitude is described as

$$F(q,s) = i\int_0^\infty b\,db\left[1-e^{i\xi(b,s)}\right]J_o(qb) \tag{2}$$

where $\xi(b,s)$ is the eikonal, b the impact parameter, J_o the zero-order Bessel function and $q^2 = -t$ the four-momentum transfer squared. The eikonal can be expressed at first order as $\xi(b,s) = \langle G_A G_B f\rangle$, where G_A and G_B are the hadronic form factors, f the averaged elementary amplitude and the brackets denote the symmetrical two-dimensional Fourier transform. Multiple-diffraction models differs of each other by the particular choice of parametrizations made for G_A and G_B and the elementary amplitude f. With this particular choice the hadronic scattering amplitude $F(q,s)$ can be studied and from it to investigate the physical observables such as σ_{tot}^{pp}, which may be evaluated from

$$\sigma_{tot}^{pp} = 4\pi\; Im\left\{F\left(q=0,s\right)\right\} \tag{3}$$

Hereafter we follow the parametrization method developed in [6] because it has the advantage of using a minimum number of free parameters: two of them (a^2,β^2) associated with the form parameters G_A and G_B and three energy-dependent parameters (C,α^{-2},λ) associated with the elementary amplitude f. According to the model

$$Im\left\{F(q=0,s)\right\} = \int_0^\infty \left[1-e^{-\Omega(b,s)}\cos\left\{\lambda\Omega(b,s)\right\}\right] b\,db\; J_o(q,b) \tag{4}$$

where the opacity $\Omega(b,s)$ is given as:

$$\Omega(b,s) = \int_o^\infty G^2\; Im\left\{f(q,s)\right\}\; J_o(q,b)\; qdq \tag{5}$$

$$\begin{aligned}\Omega(b,s) = C\{&E_1K_0(\alpha b) + E_2K_0(\beta b) + E_3K_{ei}(ab) + \\ &E_4K_{er}(ab) + b\left[E_5K_1(\alpha b) + E_6K_1(\beta b)\right]\}\end{aligned} \tag{6}$$

so, the hadron cross-section is directly determined by the expression

$$\sigma_{tot}^{pp} = 4\pi\int_o^\infty b\,db\left\{1-e^{-\Omega(b,s)}\cos\left[\lambda\Omega(b,s)\right]\right\}\; J_o(q,b) \tag{7}$$

The parameters of the model are determined by fitting the accelerator data in the interval $13.8 \leq \sqrt{s} \leq 62.5$ GeV; as it should be expected in this situation, our interpolation gives just the same predicted values as those in [6]. The main differences are, first, we estimate an error band for each of the energy dependent parameters as explained in [27,28]; secondly, we introduce new data at energies 0.546 and 1.8 TeV. Then, we proceed to extrapolate our prediction to high energies. Results are summarized in table 2 and plotted in figure 2.

The σ_{tot}^{pp} values obtained when extrapolated to ultra high energies seem to confirm the highest quoted values of the cosmic ray experiments [23,24]. That would imply the extrapolation cherished by experimentalists is wrong. In particular, the prediction given in [6] for $\sigma_{tot}^{pp} = 91.6$ mb at the Fermilab Collider energy (1.8 TeV) seems to be very high,though it should be noted that no error is quoted in that work. In table 1 we see that the measured $\sigma_{tot}^{\bar{p}p}$ at 1.8 TeV is much smaller than their prediction. It may be argued that σ_{tot}^{pp} and $\sigma_{tot}^{\bar{p}p}$ are different at high energies: This is the "Odderon hypothesis", which has been very much weakened recently [25]. Taking this into account, in our multiple-diffraction analysis it is assumed the same behaviour for σ_{tot}^{pp} and $\sigma_{tot}^{\bar{p}p}$ at high energy. Our results indicate that, if in the phenomenological multiple-diffraction approach we limit our fitting calculations to the accelerator domain $\sqrt{s} \leq 62.5$ GeV (Table 2(a)), our results are quite similar to those obtained in [6], particularly the predicted value of 91.7 mb at 1.8 TeV. Also it can be noted in Fig. 2(a) that extrapolation to ultra high energies is in complete agreement with the analysis carried out in [23] and the experimental data of the Fly's Eye [24], and even with the Akeno collaboration [19], because their quoted errors fall within the error band of our extrapolations, which in this situation turn to be as wide as the errors reported in [24]. That is, such an extrapolation produces an error band so large at cosmic ray energies that any cosmic ray results become compatible with results at accelerator energies, as it is claimed in [6]. However, if additional data at higher accelerator energies are included (Table 2(b)), therefore, the error band obviously narrows, and then things change.This can be seen in figure 2b, where we have considered data at 0.546 TeV and 1.8 TeV (76.7 mb = mean value of the E-710 and CDF Results), in which case the predicted value of σ_{tot}^{pp} from our extrapolation at $\sqrt{s} = 40$ TeV, $\sigma_{tot}^{pp} = 131.7_{-4.6}^{+4.8}$ mb is clearly incompatible with those in [23,24] by several standard deviations though no so different to the Akeno results and the predicted value in [5]. Concerning the quoted error bands, we employed the so called forecasting technique of regression analysis [26] which application in the context of hadron-hadron interactions is described in [27]: it is shown in this work that the predicted error band is highly energy-dependent, opening as energy increases and the availability of data decreases.

TABLE 2. Predicted σ_{tot}^{pp} from fitting accelerator data: (a) extrapolation with data at $\sqrt{s} \leq 62.5$ GeV [6]; (b) extrapolation including data at 546 GeV and 1.8 TeV (the two first values are interpolations). Experimental valuesare displayed in Table 1

$\sqrt{s}$ (TeV)	σ_{tot}^{pp} (mb)		σ_{tot}^{pp} (mb)	
0.55	Extrp.	$69.39^{+8.4}_{-7.4}$	Intrp.	$61.91^{+1.2}_{-1.1}$
1.8	Extrp.	$91.74^{+16.9}_{-14.7}$	Intrp.	76.78 ± 1.4
14	Extrp.	$143.86^{+38.6}_{-33.5}$	Extrp.	$110.49^{+3.2}_{-3.1}$
30	Extrp.	$167.64^{+48.9}_{-42.6}$	Extrp.	$125.63^{+4.3}_{-4.1}$
40	Extrp.	$177.23^{+53.1}_{-46.3}$	Extrp.	$131.71^{+4.8}_{-4.6}$
100	Extrp.	$210.06^{+67.6}_{-59.1}$	Extrp.	$152.45^{+6.4}_{-6.2}$
	(a)		(b)	

IV CONCLUSIONS:

It has been shown in this work that, highly confident predictions of high energy σ_{tot}^{pp} values are strongly dependent on the energy range covered by experimental data

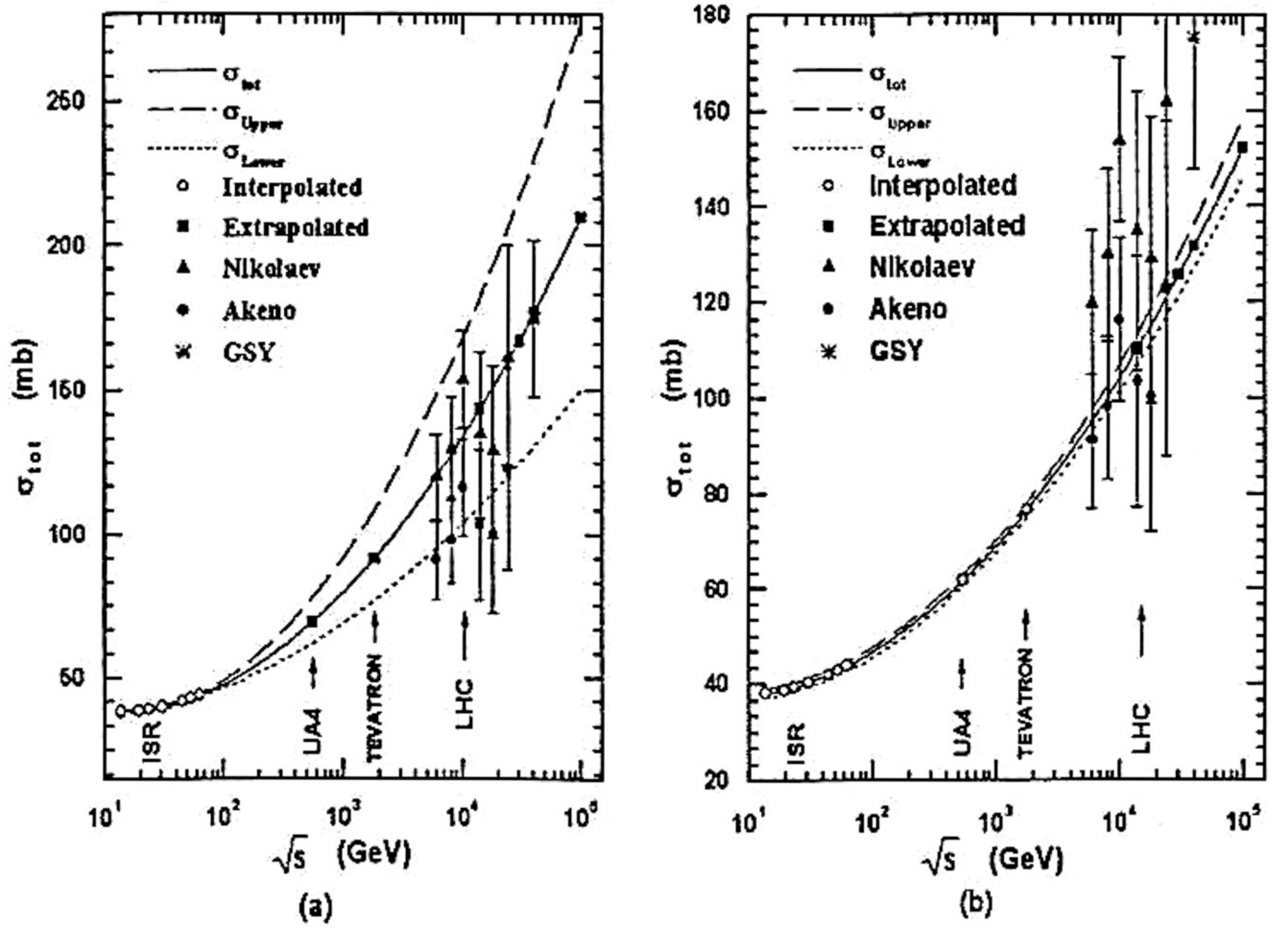

FIGURE 2. Predictions (black squares) of σ_{tot}^{pp} (a) width data at $\sqrt{s} \leq 62.5$ GeV; (b) including data at 546 GeV and 1.8 TeV. Open circles denote the interpolations.

and the available number of those data values. Accelerator data at $\sqrt{s} \leq 62.5$ GeV is nicely reproduced by our phenomenological-model calculations. In particularly, we show that if we limit our study of determining σ_{tot}^{pp} at cosmic ray energies from extrapolation of accelerator data of $\sqrt{s} \leq 62.5$ GeV, then results are compatible with most of cosmic ray experiments and other prediction models, because the predicted error band is so wide that covers their corresponding error bands (Fig. 2a). However, as the included data in our calculations extends to higher energies, that this, when all experimental available data is taking into account, the estimated values for σ_{tot}^{pp} obtained from extrapolation and those obtained from cosmic ray experiments are clearly incompatible, but compatible, within the error bars, with the Akeno results (Fig. 2b). It should be noted that our predictions are compatibles with other prediction studies [5]. Besides, predictions developed in [7] coincide with some cosmic ray data in the region around $\sqrt{s} = 30$ TeV. Taken all these convergences at face value, as indicating the most probable σ_{tot}^{pp} value, we conclude that if predictions from accelerator data are correct, hence, it should be of great help to normalize the corresponding values from cosmic ray experiments, as for instance by keeping the (k) parameter as a free one [7] instead of estimating it by complicated Monte Carlo calculations. In summary, extrapolations from accelerator data should be used to constraint cosmic ray estimations.

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