OBSERVING ULTRAHIGH ENERGY COSMIC RAYS FROM SPACE AND EARTH

International Workshop

Metepec, Puebla, México   9–12 August 2000

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AMERICAN INSTITUTE OF PHYSICS

Melville, New York, 2001

AIP CONFERENCE PROCEEDINGS ■ VOLUME 566

Prediction of σ_{pp}^{tot} at high energies with highly confident uncertainty band

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Abstract. Prediction procedures of significant physical quantities represent a useful tool in drawing inferences about the behaivor of the out-of- the range data and so, about the generator events. Theoretical predictions out of the range of a data set involve a certain degree of uncertainty. With the aim of evaluating the confidence of such predictions it is convenient to determine the uncertainty associated to the predictions of the data. In the context of p-p cross sections at very high energies a great deal of work has been done out of the energy range of accelerators using different models (single-pomeron, dipole pomeron, multiple-diffraction, QCD and so on) to extrapolate accelerator data: predictions are usually compared to cosmic ray data producing a disagreement which explanation has also been widely discussed in the literature. We claim that such comparison requires of a highly confident band of uncertainty for any parametrization model. Here, we present a statistical method that allows to determine the relevant uncertainty: predictions are developed on the basis of the multiple-diffraction model to estimate σ_{pp}^{tot} in the center of mass range $10-40\,TeV$ ($10^{17}-10^{18}eV$ *in lab*) which covers both LHC and the highest cosmic ray energies. Our study show that extrapolations without a trusful delimitation of error bands may agree with the results of Cosmic ray experiments, because experimental error bands are very large, but as soon as such a delimitation is made the predicted energy dependence of σ_{pp}^{tot} is, in general, flatter than that of cosmic ray results.

INTRODUCTION

It is well known that elastic proton-proton scattering is the most simple process in high-energy hadronic interactions. In this context a wide variety of methos have been developed for its study, ranging from phenomenological approaches up to QCD formal treatements. One of the main tasks in these studies is to detemine the total proton cross section σ_{pp}^{tot} in order to compare it with experimental data.

Total cross sections are known from accelerator experiments since the 70's, in

CP566, *Observing Ultrahigh Energy Cosmic Rays from Space and Earth,* edited by H. Salazar, et al.

the energy range $\sqrt{s} \leq 1.8$ GeV [1], [2], within the range $\sqrt{s} = 6 - 40$ TeV from Extensive Air Shower [3], [4] and from Fly's Eye Experiments [5], [6] at $\sqrt{s} = 30, 40$ Tev.

In order to know the σ_{pp}^{tot} energy behavior within the accelerator data range and beyond, it is proceeded to fit the available set data and also to predict data by extrapolation to high energies. The pioneer work with this aim was given in [7] with very good results. Actually the extrapolation is based in purely theoretical, empirical or semi-empirical methods widely accepted [8], because it is a useful tool to draw inferences about the σ_{pp}^{tot} energy behavior. However, if we analize the diverse works existing in the literature about the extrapolation problem, we find in some of them that the uncertainty associated to the σ_{pp}^{tot} prediction points is relatively large or even in other cases [9] the corresponding uncertainties associated to their predictions are not repoted. Besides, the disagreement existing between the extrapolated data to high energies from accelerator data and cosmic data, widely discussed in the literature, could be better sudied, if prediction methods would offer a confident error interval. In this preliminar work we present a statistical prediction method that allows to determine a confident statistical error interval around each of the σ_{pp}^{tot} predicted points.

I THEORETICAL FRAME

In order to illustrate the use of our statistical prediction method witin the context of p-p interactions, we must determine σ_{pp}^{tot}. There are several alternatives to do it, one of them through the Glauber's multiple difraction theory [10], under the particular approach given in [9], which has the advantage that it uses a minimum number of parameters: two a^2 and β^2 associated with the form factors G_A and G_B, and three (f,α^2 C and λ) with the elementary amplitude. Within this frame the so called opacity function $\Omega(b, s)$ is determined through the equation

$$\Omega(b, s) = \int_0^{\infty} qdq \; G^2 Imf(q, s) J_0(q, b) \tag{1}$$

Which explicit expression is

$$\begin{aligned}\Omega(b, s) = C\{&E_1 k_0(\alpha b) + E_2 k_0(\beta b) + E_3 k_{ei}(ab) + \\ &E_4 k_{er}(ab) + b\left[E_5 k_1(\alpha b) + E_6 k_1(\beta b)\right]\}\end{aligned} \tag{2}$$

Where k_0, k_1, k_{ei}, and k_{er} are the modified Bessel functions, and E_1 to E_6 are functions of the free parameters. Therefore, σ_{pp}^{tot} is determined with the following expression:

$$\sigma_{pp}^{tot} = 4\pi \int_0^{\infty} bdb \left\{1 - e^{-\Omega(b,s)} cos\left[\lambda\Omega(b, s)\right]\right\} J_0(qb) \tag{3}$$

Where b is the impact parameter, $q^2 = -t$ the four-momentum transfer squared, Jo is the zero-order Bessel function and λ is the undimensional energy-dependent parameter mentioned above. This equation was numerically evaluated, and details are described in [11].

II DESCRIPTION OF THE STATISTICAL METHOD

The validity of any statistical method to predict a given physical quantity out of the range values (extrapolation) depends on its precision to reproduce the employed data (interpolation). A fundamental task of of any prediction method is to minimize the error band of the predicted set of values. In the specific case of σ_{pp}^{tot} what is searched is to obtain a prediction beyond the energy range of the employed data with the minimum of dispersion. Among the several statistical methods to determine a confident interval around each predicted value, we use here the *Forecasting technique* [12]. This method is based on the *multiple linear regresion* theory and consists in determining a prediction equation for a quantity y (dependent variable), that depends on k independent variables (x_i), that is

$$E(y) = \sum_{i=1}^{k} \beta_i f_i(x_i) \tag{4}$$

Where f_i are arbitrary functions of x_i, and β_i are the fitting constants.

To solve the prediction problem involved in equation(4), we use the matricial formalism. Denoting with Y the matrix of $(1xn)$-dimension of the dependent variables and with X the matrix of $[nx(k+1)]$-dimension of the k independent variables, then the B matrix of the β_i fitting constant are determined through [12] $B = (X'X)^{-1}XY$, where X' denotes the transposed matrix of X and $(X'X)^{-1}$ denotes the inverse matrix of $X'X$, with those matrixes, we determine several statistical estimators, such as the Sum of Square Errors, $SSE = YY' - B'(XY')$ and the Mean Square Errors (s^2), given as $s^2 = SSE/$ (Degrees of freedom for error)

Based on these estimators we can then evaluate the uncertainty band with a $100(1-\alpha)\%$ of precision degree as follows: for prediction within the range of data by means of

$$INTB = y \pm t_{\alpha/2}^{n-p} \left\{ s^2 A'(X'X)^{-1} \right\} \tag{5}$$

for extrapolation with

$$EXTB = y \pm t_{\alpha/2}^{n-p} \left\{ s^2 \left[1 + A' \left(X'X \right)^{-1} A \right]^{1/2} \right\} \tag{6}$$

Here y denotes the central prediction corresponding to the set data included in $X-matrix$, $t_{\alpha/2}^{n-p}$ denotes the so called *the student ś t* for the n values of independent variables with p degrees of freedom, and $\alpha/2$ denotes the degree of precision. $INTB(+)$, $EXTB(+)$ and $INTB(-)$, $EXTB(-)$ denote the corresponding Upper and Lower bounds respectively. In our estimations we have used $\alpha/2 = 0.125$ which correspond to 95% of precision. The matrix A denotes the column-matrix of $1x(k+1)$ dimension, which elements $\{1, x_1, x_2, \ldots, x_k\}$ correspond to the numerical values of the β_i appearing in eq.(4). A' is the transposed matrix of A.

TABLE 1. $C(s)$, $\alpha^{-2}(s)$ and $\lambda(s)$.

$\sqrt{s}$	$C(s)$ (GeV^{-2})	$\alpha^{-2}(s)$ (GeV^{-2})	$\lambda(s)$
13.8	9.970	2.092	-0.094
19.4	10.050	2.128	0.024
23.5	10.250	2.174	0.025
30.7	10.370	2.222	0.056
44.7	10.890	2.299	0.079
52.8	11.150	2.370	0.099
62.5	11.500	2.439	0.121
546	17.4	2.790	0.193
1800	22.8	2.959	0.213

III RESULTS

Table 1 contains the energy $\sqrt{s}$, and the $C(s)$, α^{-2}(s) and $\lambda(s)$ values derived from the experimental data of σ_{pp}^{tot} [9]. The first 7 values contained in table 1 are the same of those employed in [9], whereas the last two were obtained through the method described in that reference. A second-order fitting of the values in table 1 has been obtained from the following expressions:

$$C(s) = 10.919 - 0.648\ ln(s) + 0.085\ ln(s)^2 \tag{7}$$

$$\alpha^{-2} = 2.311 - 0.137\ ln(s) + 0.0181\ ln(s)^2 \tag{8}$$

$$\lambda(s) = 0.2486 - 0.073\ e^{-ln(s/400)/1.19} - 0.176\ e^{-ln(s/400)/5.698} \tag{9}$$

Using the procedure described through equations (4)-(6), we have determined the Upper and Lower bounds [eqs. (5) and (6)] for $C(s)$ and $\alpha^{-2}(s)$ for each one of the energy values. The results are shown through figures 1(a) and 1(b). In order to render compatible eq.(9) with the mentioned procedure we have done a linear fitting of the λ values. The specific determination of the uncertainty associated with σ_{pp}^{tot} values was indirectly done by the previous evaluation of the corresponding Upper and Lower bounds of the energy-dependent parameters (C, α, λ). The obtained results are shown through figures 2(a) and 2(b).

IV ANALYSIS AND CONCLUSIONS

The analysis of figures (1-2) shows that as data becomes more scarse, as it happens as energy increases, the uncertainty interval becomes wider, which indicates an information lost in the predicted values; obviously, this lost is reflected on the σ_{pp}^{tot} predicted values. We can observe in figures 1(a) and 1(b) that he difference between both curves, the Martini & Menon [9](hereafter M & M) curve and ours (with 546 and 1800 Gev data) is increased as the energy is increased: this difference can be seen as a measure of the precision of our results. In figure 2(a) occurs a similar behavior, as energy is increased the uncertainty associated to the predicted σ_{pp}^{tot} values also is increased; it can also be observed that there is not difference between the M & M predicted values and the prediction of this work (the solid line). Is notorious in figure 2(a) that the uncertainty is very large at high energies but, if more points are added to the prediction process (e.g. at energies 0.546 and 1.8 TeV) the uncertainty is decreased, figure 2(b), so, in $\sqrt{s} = 40TeV$, we obtain an uncertainty interval with $\pm 4.6mb$ which is much less that those deduced through other methods (i.e. [3]). Figure 2(a) shows the interpolations of M & M and those of this work (solid line) employing only the ISR accelerator data. Figure 2(b) shows the interpolation employing all available accelerator data (white boxes) and extrapolations (black squares); The dashed line corresponds to the central curve of

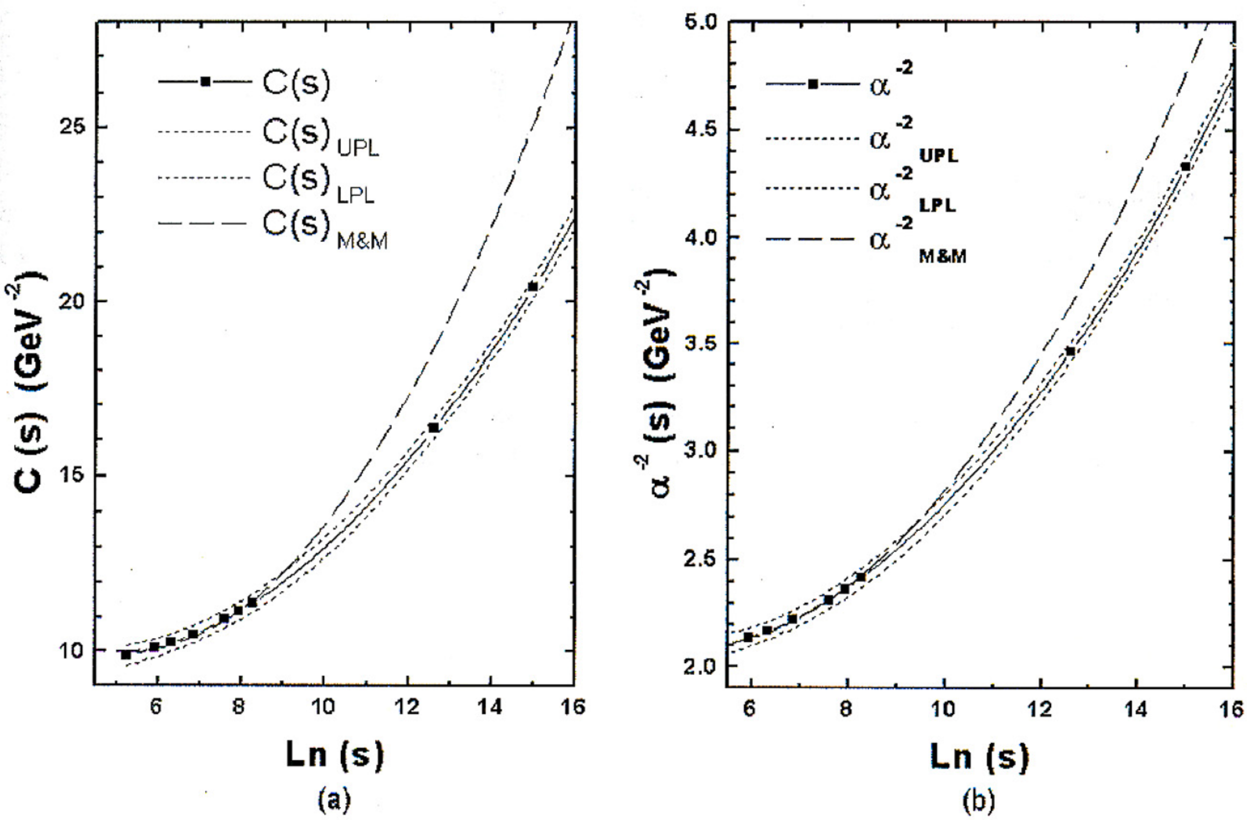

FIGURE 1. Our fittings (black squares) with data at 0.546 & 1.8 TeV and M & M fittings (dashed curve) with Upper and lower bounds for: (a) $C(s)$, and (b) α^{-2}.

Fig. 2(a) when only the ISR data is considered. The corresponding values of σ_{pp}^{tot} in Figures 2(a) and 2(b), with their respective uncertainty bands are displayed in Table 2 of [11]. We conclude that the employement of a trusful statistical method to delimitate highly confident uncertainty bands leads to obtain highly precise extrapolation results. The use of the *Forecasting method* in the specific case of σ_{pp}^{tot}, seems to indicate that the high energy values of σ_{pp}^{tot} are lower than the obtained with Cosmic Ray expeiments, that is, the energy dependence might be flatter than that infered from Cosmic Rays. A relevant discussion about this implication of our results is given in [11].

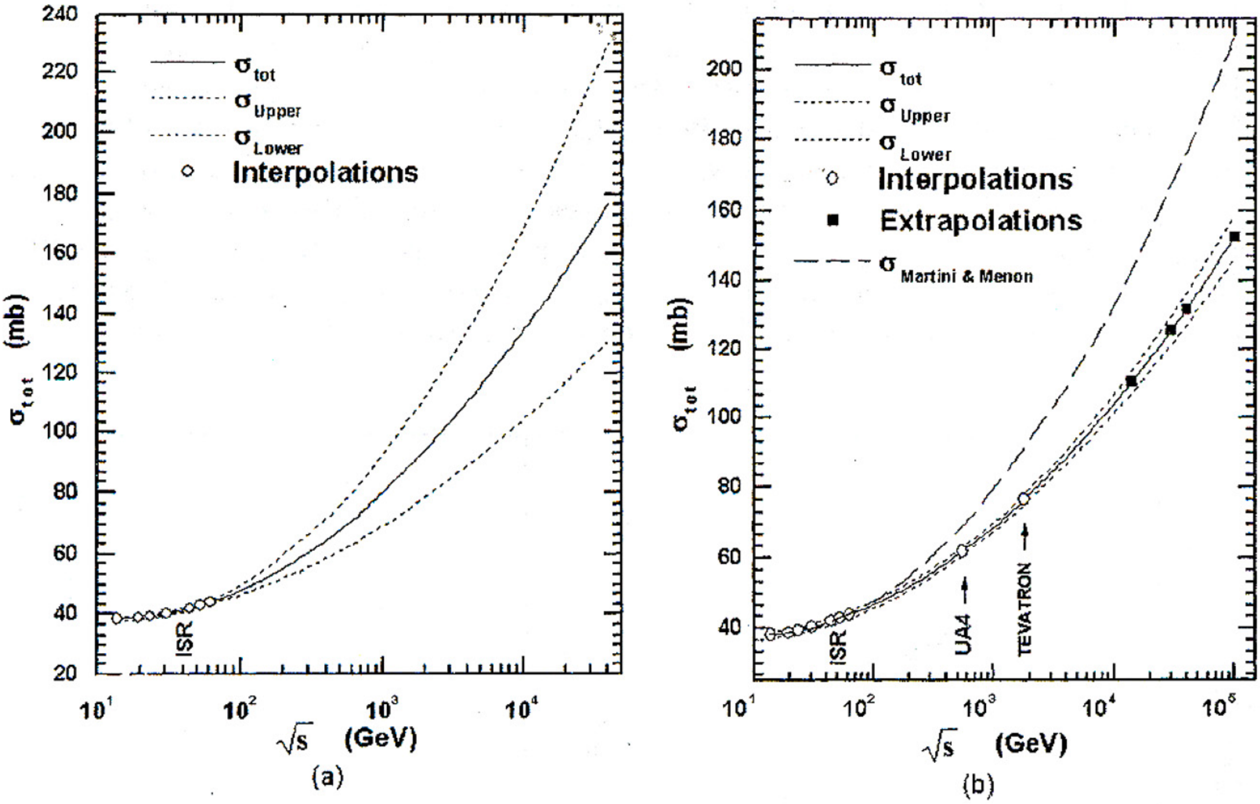

FIGURE 2. Predictions of σ_{tot}^{pp} values (a)employing only the ISR accelerator data, (b)Interpolation and extrapolation employing all available accelerator data.

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