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CONTRIBUTION OF ENERGY FLUCTUATIONS IN THE EVOLUTION EQUATION FOR COSMIC RAY SPECTRA

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Abstract

For the derivation of particle energy spectra from the continuity equation of the Fokker Planck-type in energy space, it is usually neglected the term of fluctuations in the rate of energy change, under the assumption of its smallness. We show here for statistical Fermi acceleration and neutral current sheet acceleration under, three different scenarios, that the consideration of fluctuations in stationary and non-stationary conditions leads to qualitative and quantitative modifications of the energy spectrum.

1. Introduction. It has been established long ago that electromagnetic interactions of charged particles have in general a statistical behavior, in the sense that collisions of individual particles are independent events. In the case of particle energy loss by interaction with matter, magnetic or photonic fields, there are in some extent deviations from an average value of energy loss per collision; that is, particles of the same kind and energy do not loss exactly the same amount of energy in traversing matter, or a given electromagnetic field. In the particular case of coulomb collision and synchrotron energy losses of electrons, these statistical fluctuations in the energy loss per collision are relatively small (e.g. Rossi, 1952), so that an appreciable energy change is produced only if the number of collisions is very high. Concerning energy gain, the acceleration may be of regular (systematic), or, statistical nature. In the case of statistical Fermi acceleration it is well known (e.g. Ginzburg and Syrovatskii 1964) that even if on the average there is no acceleration (dE/dt)=0, because the magnetic field does not increase on the average over the whole acceleration volume, however, fast particles may still be produced because the deviation of the local physical parameters from their average behavior, which in turn is translated in corresponding deviation of the acceleration efficiency from the average value: for instance, fluctuations in density, or even if the whole magnetic field strength remains constant particles may pass through regions of increasing and decreasing magnetic fields in time or direction, which is translated in fluctuations of the Alfvén hydromagnetic velocity, u, of the accelerating scatter centers. Therefore, particles of the same kind and energy may collide with inhomogeneities of different u. Also, if the distribution of scatter centers is at random, particles of the same energy may differ in mean free path - ℓ(the characteristic acceleration step) and so, in their remaining time, τ, in the region, from the average values which characterize the process. These fluctuations in the physical parameters of particle sources, and consecuently in the acceleration process is translated in fluctuations around the average energy gain rate, which in the Fermi statistical mechanism is given as $(dE/dt)=\xi\alpha\beta E$, where the acceleration efficiency $\alpha=u^2/c\ell=u^2/cv\tau$, and β is the particle velocity, v, in terms of the light velocity, c; $\xi\sim 1$ (if all the collisions are of head-on - type $\xi\sim 2$, and if collisions are with spherical elastically scattering centers $\xi\sim 1.33$), and E is the total energy. In the case of neutral current sheet acceleration, we are in principle dealing with a deterministic process, where no noticeable deviations from the average energy change rate $(dE/dt)=K_1\beta$ is expected, because the accelerating electric field $\vec{\mathcal{E}}=(1/c)\vec{V}_d X\vec{B}\sim$ cte., in the measure that as the magnetic field strength $\vec{B}$ decreases toward the neutral line, there is a compensating effect by the increase of the incoming defrozen plasma velocity, - $\vec{V}_d$; $K_1=qec\varepsilon=2.89X10^{10}\varepsilon(eV/s)$. However, in the more strict sense, particles of the same kind and energy, but in different side of the neutral line, find magnetic - field of different direction, and a given particle traversing the neutral line

*ON LEAVE FROM THE INSTITUTO DE GEOFISICA, UNAM finds in one side a decreasing and in the other an increasing magnetic field, each one of apposite polarity. Though the electric field may be considered as constant, in average, it is realized that some deviations must take place. Nevertheless, which is true is that in some specific topologies of neutral current sheets (Petschek,1964), a substantial volume of the sheet is populated by fluctuating electric fields, around the average value, where stochastic acceleration becomes predominant over deterministic acceleration. Since whatever the involved acceleration mechanism, the magnitude of fluctuations in the energy change of particles is proportional to the magnitude of the energetic transfer between particles and the medium, it must be expected that in very energetic events, as stellar (solar) flares, fluctuations play an important role in the formation of the energy spectrum and in the regulation of its shape and magnitude. At this regard, for the particular case of statistical Fermi acceleration, under stationary conditions, it has been argued (Ginzburg and Syrovatskii,1964), that consideration of fluctuations do not lead to any qualitative modification of the galactic cosmic ray spectrum relative to the spectrum when fluctuations are neglected.

In this work, we show that when specific scenarios and definited acceleration parameters are considered, fluctuations lead to qualitative and quantitative modifications of the spectrum, whatever the scenario and acceleration process, in both stationary and non-stationary conditions.

2. Solutions of the evolution equation with fluctuations. The description of Cosmic Ray evolution is generally made by a Fokker-Planck-type continuity equation, which general form has been extensively discused (e.g. Ginzburg,1958). When spatial diffusion and catastrophic particle disparitions or apparitions (such as chemical transformations) are neglected, the equation becomes:

$$\frac{\partial N(E,t)}{\partial t} + \frac{\partial}{\partial E}\left[\frac{dE}{dt}N(E,t)\right] - \frac{1}{2}\frac{\partial^2}{\partial E^2}\left[D(E)N(E,t)\right] + \frac{N(E,t)}{\tau} = q(E,t) \,, \qquad (1)$$

where the dependence in position has been neglected under the assumption of spatial homogeneity of particles in the source. $N(E,t)$ represents the number of particles of a given kind per total energy interval at a given time t. The 2^{nd} term represents the average systematic energy change, the 3^{rd} term is the fluctuation in particle energy, the 4^{th} term is the escape of particles from the source at a rate τ^{-1}; where τ is the mean life in the acceleration region, and the term at the right side is the injection into the acceleration process. The fluctuation coefficient in the case of the statistical Fermi mechanism $D(E)= 2\xi\alpha\beta^3E^2$, where α is the average acceleration efficiency, whereas for electric field acceleration in neutral current sheets, $D(E)=\zeta cq^2e^2\ell\varepsilon^2\beta=K_2\beta$, where $K_2=2.8X10^{10}\ell\zeta\varepsilon^2(eV^2/s)$, and ℓ is the average length along which the average electric field ε is operating - here $\zeta\sim1$, and in analogy with the Fermi process, when particles in a gyrocycle along the $V_d XB$ direction moves toward the neutral line, there are "overtaking" interaction with the magnetic field, whereas when movement is against the neutral line there are "head-on" interactions. So, $D(E)=\delta' q^2e^2\varepsilon^2\beta$, where $\delta'=\zeta\omega$, with - $\omega=c\ell$, while for Fermi acceleration $D(E)=2\delta\beta^3E^2$ with $\delta=\xi\alpha$. To solve (1), three typical scenario for cosmic ray production are chosen, which were extensively described in Pérez-Peraza and Gallegos (1987): the first, when there is only one acceleration phase of the background thermal population up to high energies. In the 2^{nd} scenarios there are two acceleration stages, an injection process from thermal energies where the (NCS) acceleration process is used and a secondary - stage where the acceleration process takes only particles of the injection process above a certain threshold energy value corresponding to the hydromagnetic velocity. The third scenario is similar to the 2^{nd} one, with the difference that the threshold value is the local thermal energy ~ 0.5 kT, so that in addition of the injected particles from N.C.S. acceleration, also particles of the local -- background participate to the second acceleration stage. The stationary and non-stationary solutions of (1), when the fluctuation term is neglected was given by Pérez-Peraza and Gallegos (1987), for the three scenarios under consideration, solution of (1) when $\beta\to1$ is of the form:

$$N(E,t)=(\Delta/4\pi t)^{0.5}\exp\{-t/\tau-(G-\Delta)^2t/\Delta\}\int_{E_o}^{\infty} N(E',0)\exp\{-(x-x')^2/4\Delta t\}dE'' \quad +$$

$$\int_0^t (\Delta/4\pi t')^{0.5}\exp\{-t'/\tau-(G-\Delta)^2t'/\Delta\}\int_{E_o}^{\infty} q(E')\exp\{-(x-x')^2/4\Delta t'\}dE''dt', \qquad (2)$$

where $x=\ell nE, G=\alpha, \Delta=\delta$ for Fermi acceleration, while $G=\omega$, $\Delta=\delta'$ for (NCS) acceleration, E' appears from $t=\int_{E'}^{E} dE''/(dE''/dt)$. The threshold value E_o may eventually be different in the 1^{st} and 2^{nd} term of (2) according to requirements of the scenario. The stationary solution of (1) is given as:

$$N(E)=C_1e^{\gamma_1 x} + c_2e^{\gamma_2 x} + (e^{\gamma_1 x}/\gamma_1-\gamma_2)\int_{E_o}^{E} q(E'')e^{-\gamma_1 x}dE'' + e^{\gamma_2 x}/(\gamma_2-\gamma_1)\int_{E_o}^{E} q(E'')e^{-\gamma_2 x}dE'' \qquad (3)$$

where $\gamma_{1,2}=(a/2)\{-1 \pm (1-4b/a^2)^{0.5}\}$ with $a=4-2B/\Delta$ and $b=2(1-B\Delta-1/\Delta\tau)$. The C_1 and C_2 constants are respectively evaluated from the frontier conditions, given by the system of equations defined when N(E) in (3) is setted to the Maxwell distribution evaluated in 0.5 kT, $N(E_{th})=\{2\pi N_o/(\pi kT)^{1.5}\}(E_{th}-mc^2)^{0.5}e^{-1.5}$, where N_o is the total number of particles participating in the process, as defined in Pérez-Peraza and Gallegos (1987), and on the other hand setting $N(E)\equiv 0$, at $E=E_m$, where E_m is the high energy cutoff of the acceleration process. In the specific case of our scenarios, we have that in the 1^{st} one, there is no external injection, so that the 2^{nd} term in (2), and the 3^{rd} and 4^{th} terms in (3) disapear. The constants are $C_1=N(E_{th})/(E_{th}^{\gamma_1}-E_{th}^{\gamma_2} E_m^{\gamma_1-\gamma_2})$ and $C_2=-C_1E_m^{\gamma_1-\gamma_2}$; for the Fermi process N_o in $N(E_{th})$ is taken from the Alfvén velocity up to ∞, and for (NCS) acceleration N_o is taken from 0.5 kT up to ∞. Fig. 1 shows the confrontation of the spectrum with and without fluctuations, with Fermi acceleration of electrons, where $\alpha=0.2s^{-1}$, $n=10^8cm^{-3}$, $\tau=0.8s$, $\xi=1.33$ and $E_m=10$ MeV for the stationary case, and $\alpha=2s^{-1}$, $n=10^9cm^{-3}$, $\tau=0.06s$, $E_m=6$ MeV, $\xi=0.2$ for the non-stationary solution; for this later case we used in (2) $E_o=0.5kT$, $N(E',0)$ is the Maxwell distribution with N_o evaluated according to the acceleration mechanism. Within the frame of the 2^{nd} scenario, where no thermal particles participate to the acceleration, the 1^{st} term in (2) and the 1^{st} and 2^{nd} term in (3) disapear. The injection process has been assimilated to neutral current sheet acceleration, due to its impulsive nature; according to Pérez-Peraza and Gallegos (1987), the spectrum from this process may be written as $q(E)=A'e^{-C_3E}$, where $A'=N_o/\tau K_1$ and $C_3=1/K_1\tau$. The solution in the stationary case is $N(E)=Ae^{-C_3E}$ where $A=(A'/\delta')\{(f+\gamma_2+1)/(\gamma_1-\gamma_2)(f+\gamma_1)(f+\gamma_2)\}$ and $f=1/kT$. In both solutions (1) and (2) E_o is the corre-

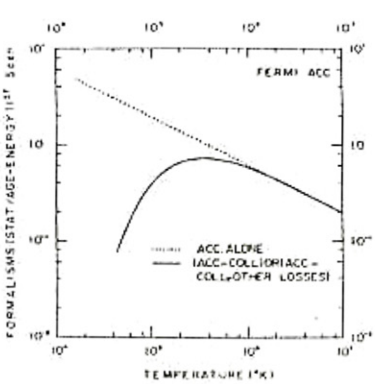

FIG 1

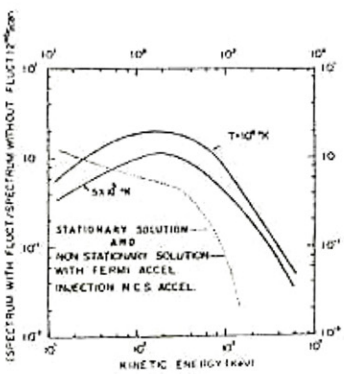

FIG 2

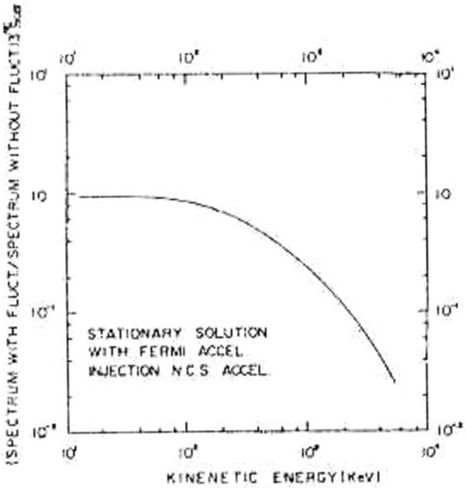

FIG 3

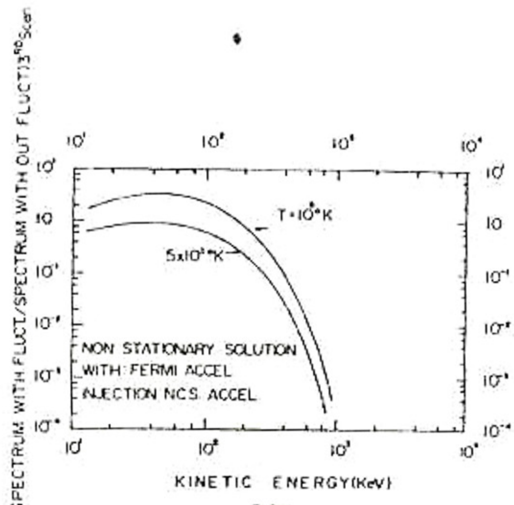

FIG 4

sponding energy to the local hydromagnetic velocity. Fig. 2 shows the stationary solution with $\varepsilon=2\times10^{-4}$ V/cm, $\zeta=1$, $\tau=0.1$s and $n=10^{11}$cm^{-3} for the injection and $\alpha=0.23s^{-1}$, $\tau=0.6$s, $\xi=2$, $n=10^{8}$cm^{-3}, $E_m=10$ MeV for acceleration. For the non-stationary case $\varepsilon=1.8\times10^{-6}$ V/cm, $\zeta=1$, $\tau=0.08$s, $n=10^{11}$cm^{-3} for injection, and $\alpha=2s^{-1}$, $\xi=0.5$, $\tau=0.1$s, $n=10^{8}$cm^{-3}, $E_m=10$ MeV. Within the frame of the 3rd scenario, ($N(E)\neq0$; $q(E)\neq0$) where N(E), in the stationary solution, is given by the two first terms of (3); E_o was taken as 0.5kT. The stationary and non-stationary solutions are obtained by addition of the corresponding solutions in the 1st and 2nd scenarios. Fig. 3 shows the stationary solution with $\varepsilon=10^{-4}$V/cm, $\zeta=1$, $\tau=0.1$s, $n=10^{11}$cm^{-3} for injection, and $\alpha=0.2s^{-1}$, $\xi=0.2$s, $\tau=0.7$s, $n=10^{8}$cm^{-3}, $E_m=10$ MeV for acceleration. Fig. 4 shows the non stationary solution with $\varepsilon=1.8\times10^{-5}$ V/cm, $\zeta=1$ $\tau=0.05$s, $n=10^{11}$cm^{-3}, and $\alpha=1.2s^{-1}$, $\xi=2$ $\tau=0.1$s, $n=10^{8}$cm^{-3}, $E_m=10$ MeV.

3. Results and Conclusions. It can be appreciated on Figs. (1) to (4) that the general effect of fluctuations on the energy spectrum is a particle depression at high energies, with the exception of the non-stationary solutions in the 1st scenario, for $T>10^{5}$°k. This may be interpreted as with fluctuations in the magnetic field and density in the medium, the corresponding fluctuations in the hydromagnetic velocity u entails is some way a variation of the mean confinement time in the form $\tau\sim(1/E)^{\eta}$, even if in average α remains constant around its average value. The non-stationary treatment shows that the effect of fluctuations is very sensible to the acceleration parameters, source temperature and in some extent, to the chosen scenario. Though the effect of the fluctuations on stationary conditions also depends on the parameters of the process, it is independent on T with similar behavior in different scenarios: i.e. particle depression with energy increase. Therefore, we conclude that fluctuations affect the spectrum as well in quantitative as in qualitative way, though the degree of such effects is widely assorted. To illustrate this asseveration, let analyse the simplest case, that of the stationary solution in the 1st scenario. We show that in this case $N(E)\sim C_1 e^{\gamma_1 X}$ or $N(E)\sim C_2 e^{\gamma_2 X}$, with $\gamma_{1,2}=(\alpha/\delta-2)\pm\{(2-\alpha/\delta)^2-2(1-\alpha/\delta-1/\delta\tau)\}^{0.5}$ (with $\xi=2$) for the Fermi process: it can be appreciated that if $\delta\tau>>1$, we obtain $N(E)\sim C_2 E^{-2.6}$, which is just of the order of the galactic Cosmic Ray spectrum, whereas in the same situation without fluctuations, when $\delta\tau=2\alpha\tau>>1$ we have $\gamma=1+1/\alpha\tau=1+2/\delta\tau\simeq1$; therefore the effects of fluctuations - gives a completely different qualitative description in opposition to what is conventionally argued (e.g. Ginzburg and Syrovatskii, 1964). With fluctuations and $\xi<2$, the product $\delta\tau$ must be comparatively higher, in order to obtain $\gamma\sim2.6$. For a given value of ξ it is direct to set limits in the value of $\alpha\tau$ and so to infer about α and τ, as well in solar as in Galactic Cosmic Rays.

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