



# Determination of GLE of Solar Energetic Particles by Means of Spectral Analysis

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Received 2019 February 23; revised 2019 April 20; accepted 2019 May 15; published 2019 June 25

## Abstract

Using three nonstationary solar series, the solar flare index (FS), the sunspots index (SS), and the solar flux (F10.7) index, we apply the Morlet wavelet analysis to determine the most dominant harmonics of solar activity, 1.73, 3.27, 4.9, 10.4, and 11 yr. The periodicities obtained are processed by the fuzzy logic method, which allows us to reproduce the occurrence dates of ground level enhancements (GLE), since 1942–2006, which we use as a training baseline of these spectral techniques to determine the occurrence of solar particle enhancements in solar cycles. Then, the result of fuzzy logic is extended to periods later than the training period so as to cover the end of cycle 24 and the beginning of cycle 25. In addition to the forecastable aspect of this work, the obtained results are of high interest in view of the recent controversy that has arisen in relation to the occurrence of small GLE (namely sub-GLE), during cycle 24.

*Key words:* Sun: flares – Sun: particle emission

## 1. Introduction

The solar energy particles that arrive to the ground have been given several names: relativistic solar proton events and/or ground level enhancements (GLE). The latter name has remained the general one, and a subdivision has even been presupposed. The GLE are measured at the terrestrial level by the worldwide network of neutron monitor (MN) detectors. These sporadic events are associated with solar flares and are assumed to be of a solar quasi-stochastic nature: their occurrence is not always connotative of solar activity intensity. Taking into account that even when solar cycle 22 was much more intense than cycle 23, the latter had more GLE than cycle 22; for example, there were 13 GLEs in the period from 1989 July to 1991 June, and not a single event since the end of 2006 December up to 2012 May. A previous study that establishes the synchronization between some periodicities of the various layers of the solar atmosphere argues against a complete stochasticity of the relativistic particle production phenomenon. This leads to the determination of precursors that are not seen in the galactic cosmic radiation outside the periods of GLE occurrences (Pérez-Peraza et al. 2009). Such synchronization seems to indicate that the production of GLE is not an isolated local phenomenon, but rather it involves global regions of the Sun's atmosphere. In this last study, it was shown that despite the quasi-stochastic nature of GLEs, it is possible to predict them with relative precision, months or even years before they occur: even for the next solar cycle. Additionally, in this work, we can clearly distinguish the occurrence of 10 GLE during solar cycle 24 that had not been comprehensively categorized.

## 2. Data and Methodology

### 2.1. Morlet Wavelet Analysis

To determine the main oscillation periodicities as well as their time evolution in nonstationary series, such as those of solar energetic particles, we apply the Morlet wavelet technique (Torrence & Compo 1998). This is a very well-known tool for analyzing localized variations of power within a given time series for many different periodicities when one is dealing with a nonstationary series and the coherence between two

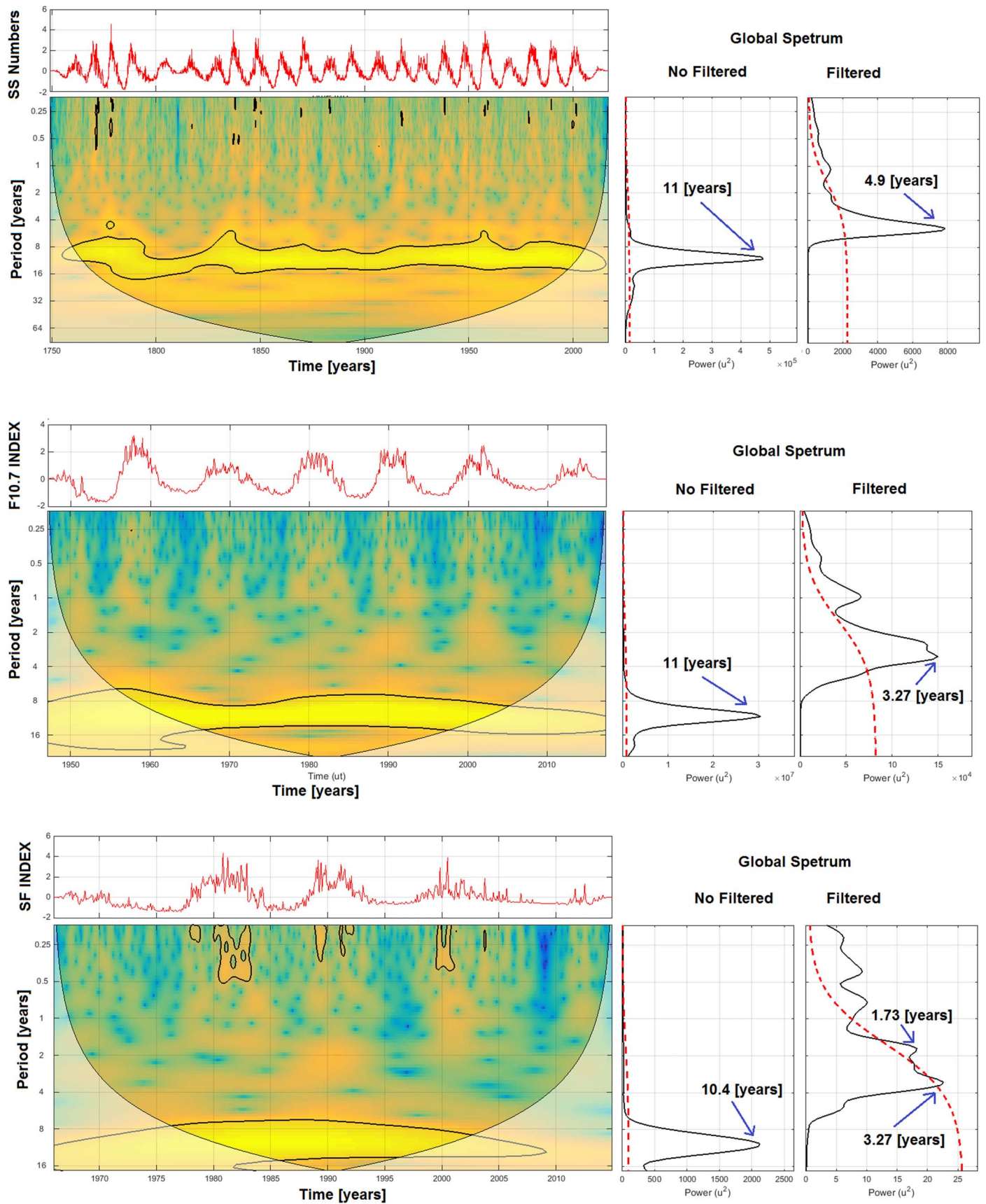
nonstationary series. The so-called global wavelet spectrum (GWS) is an average of the power spectra at each resolution level. That is to say, it is assumed that the time series has an average power spectrum relative to the red noise of the Fourier series: harmonics above this average spectrum (the slashed line in the right panels of Figure 1) represent real signals with levels of reliability higher than 95%. The importance of the GWS is in the distribution of signals with the same characteristics in order to determine which harmonics contain greater power (Torrence & Webster 1999).

We apply the wavelet analysis to the series of monthly data obtained from the following index that pertains to solar activity: number of sunspots (SS) from 1749 to 2017 (<http://www.sidc.be/silso/datafiles>), solar flux index (F10.7) from 1947 to 2017 (<https://www.esrl.noaa.gov/psd/data/correlation/solar.data>; Xiao et al. 2017; Chatterjee 2001; Henney et al. 2012), and the Flare Index (FI) from 1966 to 2014 ([http://www.koeri.boun.edu.tr/astronomy/fi\\_nedir.htm](http://www.koeri.boun.edu.tr/astronomy/fi_nedir.htm); Ataç & Özgüç 1998, 2001). Figure 1 demonstrates the wavelet spectrum and the global energy spectrum (intermediate panels) of each of the series. In order to discern high frequencies, we apply the Daubechies filter (Daubechies 1992) so as to eliminate the 11 yr harmonic or its approximate in the case of Index F10.7, that contains a much higher level of energy and thus conceals the shorter periods.

The dominant periodicities that are present in different series that we use in our analysis refer to sunspots: 11 and 4.9 yr; for the index F10.7: 11 and 3.27 yr; and for FI: 10.4, 3.27 and 1.73 yr. It should be mentioned that in the case of the FI, we found the quasi-biennial periodicity (1.73 yr) proposed by Velasco Herrera et al. (2018).

### 2.2. Fuzzy Logic

The procedure of Fuzzy Logic consists of calculating the time intervals of occurrence of the GLEs, by means of creating membership functions (MFs) for the selected periodicities of greater energy, in the wave power spectrum of the studied series, as described by Mendel (1995). We observe that the amplitude of the dominant periodicities, and their behavior during the occurrence of a GLE event, have similar



**Figure 1.** Spectral analysis: the upper panel of the first box shows the time series of the number of sunspots, the central panel of the box refers to the wavelet spectrum, and the right panel is its global energy spectrum before and after filtering. Similarly, in the central box we have the F10.7 series and below, the corresponding Wavelet and its global spectrum. Similarly, in the lower panel the corresponding information of the FI can be found.

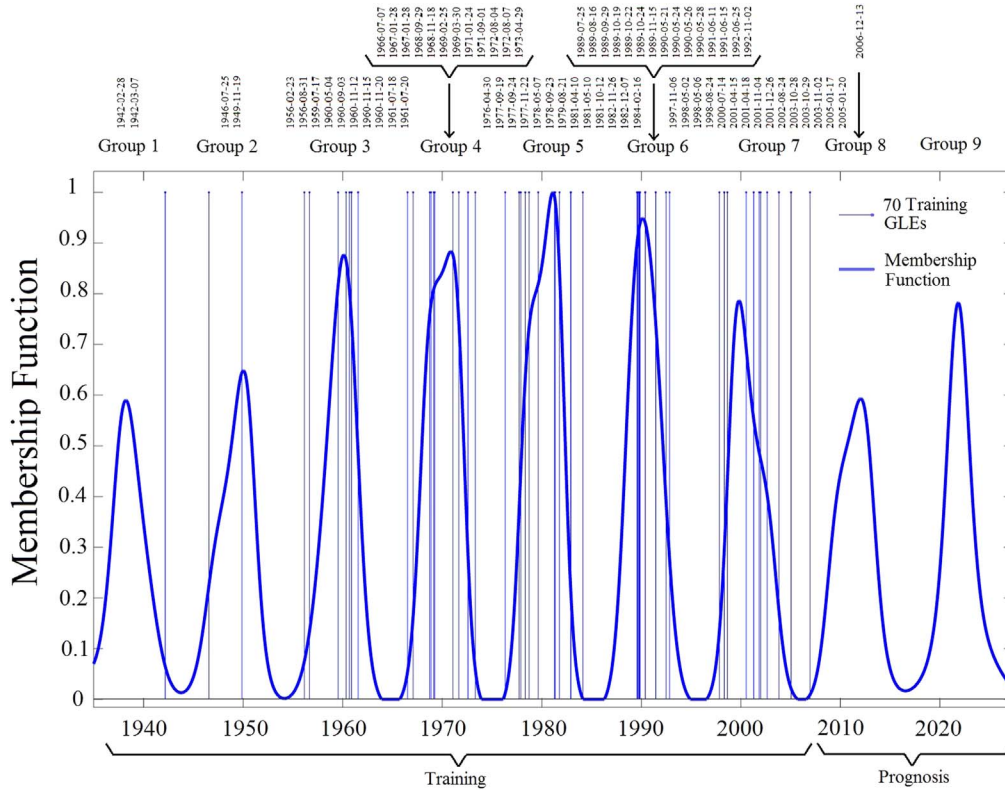


Figure 2. Fuzzy Logic results of the 70 training GLE.

characteristics that allow us to estimate the time intervals in which subsequent events may occur in the future.

The MFs are usually built according to criteria of experts in the area of study or, alternatively, these can be calculated by using mathematical data analysis algorithms, which are applied mainly in the Theory of Control Systems (e.g., Chuen 1990). In our case, the MF is the curve that describes the degree to which an element of the set of amplitudes of a certain periodicity in the 70 GLEs belongs. The concept of fuzzy logic emerges from the fact that an MF can describe the occurrence of a given GLE. In our analysis, the MF was constructed with the product of the equations of two standard Gaussian curves expressed in Equation (1): the mean and the standard deviation are obtained with the data and the amplitudes of the frequency and its derivate during the occurrence of the studied events.

$$\mu_A = \frac{1}{\alpha_A \sqrt{2\pi}} e^{-\frac{(t-\beta_A)^2}{2\beta_A^2}} \times \frac{1}{\alpha_{dA} \sqrt{2\pi}} e^{-\frac{(t-\beta_{dA})^2}{2\beta_{dA}^2}}. \quad (1)$$

Equation (1) represents the function of the membership of the frequencies, according to the studied periodicities,  $\alpha_A$  and  $\beta_A$ , that represent the average and the standard deviation of the frequency amplitudes, respectively;  $\alpha_{dA}$  and  $\beta_{dA}$ , which are calculated from the amplitude of the derivative of periodicity; in both cases, the average and the standard deviation are calculated with the data of the amplitudes of the frequency at the moment in which the events of interest (or training) occurred in the past, that is, the known GLE.

Finally,  $t$  is the variable that represents the distribution of the amplitudes of the frequencies. Therefore, although the proposal of an MF is somewhat arbitrary, in the sense of selecting the equation of a Gaussian function, and not of another function,

Table 1  
First GLE and Last GLE of the First Eight Groups

Group	First GLE	Last GLE
1	***	(1942 Mar 7)
2	(1946 Jul 25)	(1949 Nov 19)
3	(1956 Feb 23)	(1961 Jul 20)
4	(1966 Jul 7)	(1973 Apr 29)
5	(1976 Apr 30)	(1984 Feb 16)
6	(1989 Jul 25)	(1992 Nov 02)
7	(1997 Nov 6)	(2005 Jan 20)
8	(2006 Dec 13)	***

Table 2  
Intervals Calculated for the Two Subgroups of GLE: The First and the Last of Each Group

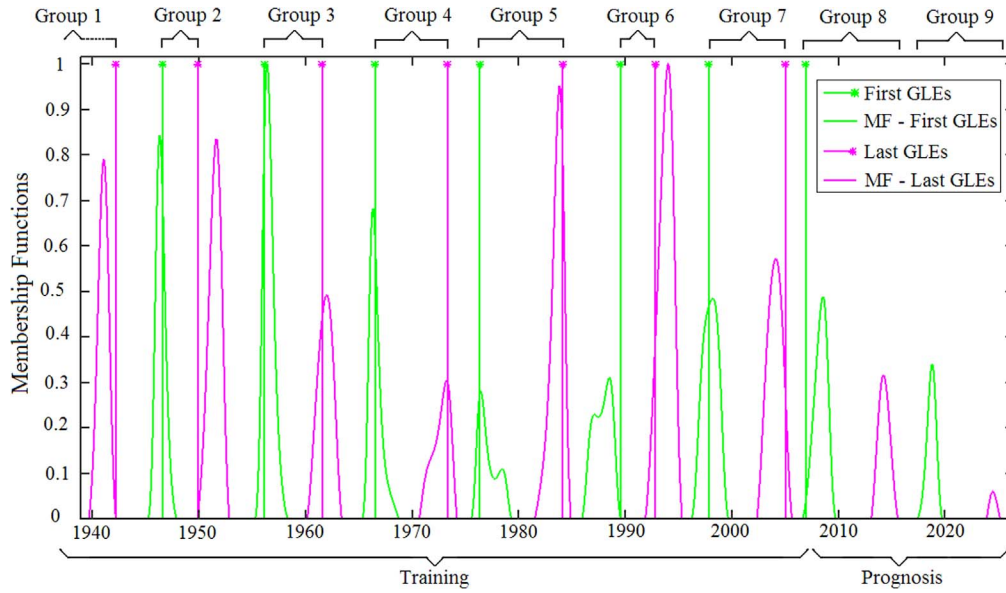
Group	Begin	End
8	2006 Aug 26	2015 Aug 3
9	2017 Jun 13	2005 Mar 5

we assume that our data can approximate a Gaussian bell, and the MF is statistically related to our data.

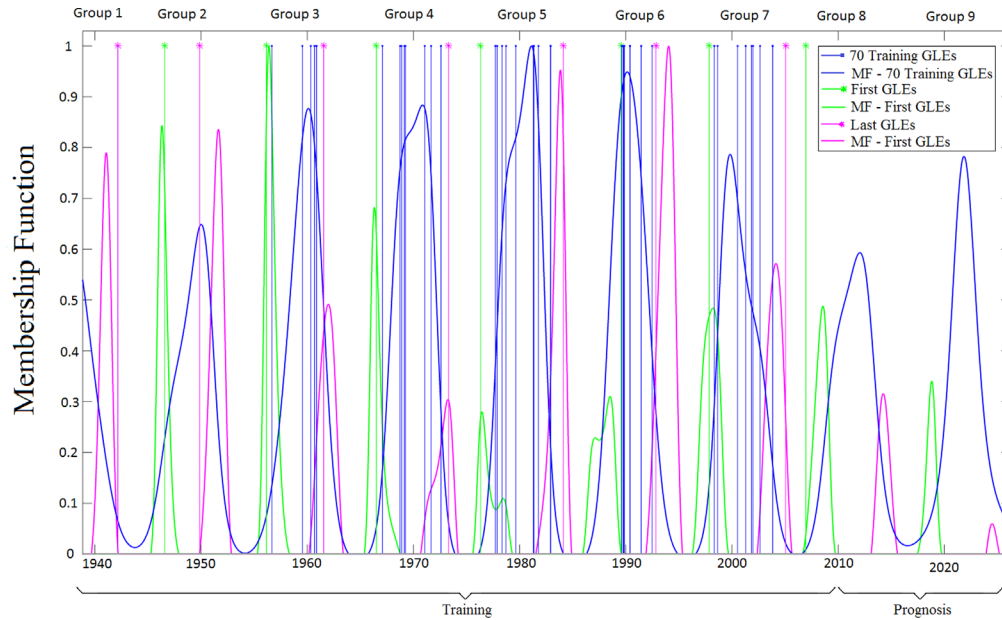
Figure 2 shows the MF constructed with the 70 training GLE mentioned. By definition, MFs have maximum unit amplitude, and 0 indicates that there is no membership (Mendel 1995).

To predict the amplitude of the MF, we base our calculations on the prospective behavior (periodic behavior in the future) of the amplitude of a certain frequency. The information of the MFs calculated for all analyzed periodicities leads us to define time intervals for the probable occurrence of an event. Once the MFs for each frequency are constructed, the next step is to calculate all of its intersections, Equation (2), which results in





**Figure 3.** Membership Functions of the first GLE and last GLE subgroups.



**Figure 4.** Membership function graph of the initial 70 GLE training, and the membership functions of the first GLE and the last GLE subgroups.

the product.

$$\Pi = \mu_A \cap B \cap C \cap \dots = \mu_A \times \mu_B \times \mu_C \times \dots, \quad (2)$$

where  $\mu_A \cap B \cap C \cap \dots$  denotes the intersection function and  $\mu_A$ ,  $\mu_B$ ,  $\mu_C$ , ... the MFs of each of the selected periodicities.

Figure 2 shows the results of fuzzy logic: the 70 GLE training events are illustrated in light blue. The resulting MF is in dark blue.

In this work we continue with the previous assumptions by using the behavior of periodicities to describe the occurrence of the GLE. The behavioral characteristics of periodicities determine the time intervals in which a GLE may occur. The procedure for calculating time intervals is to create MFs for the periodicities with higher energy in the wavelet power spectra, of the three indexes. Unlike to previous work, Pérez-Peraza & Juárez-Zuñiga (2015), for “training” purposes the GLE were

grouped into three categories: first, last, and intermediate, which were previously classified on the basis of the 11 yr solar cycle. Later, the MF of each of those three groups was obtained. On this basis the GLE occurrence intervals were determined.

Instead, in this work, we use the first 70 GLEs (from 1942 to 2006) as initial training data, and together with the seven periodicities (Figure 1 and Equation (2)) we obtained a more accurate MF. From this MF the first and last subgroups were obtained, see Table 1.

Nine groups are formed from the 70 GLE training. Thus we call the training zone the first seven groups and the beginning of the eighth group. We call the rest of the eighth group and the entire ninth group the prognosis zone.

In Figure 2, we show the harmonic behavior of the dates of occurrence of the GLE, according to the selected periodicities of the indexes worked on (Equation (2)). This result is in

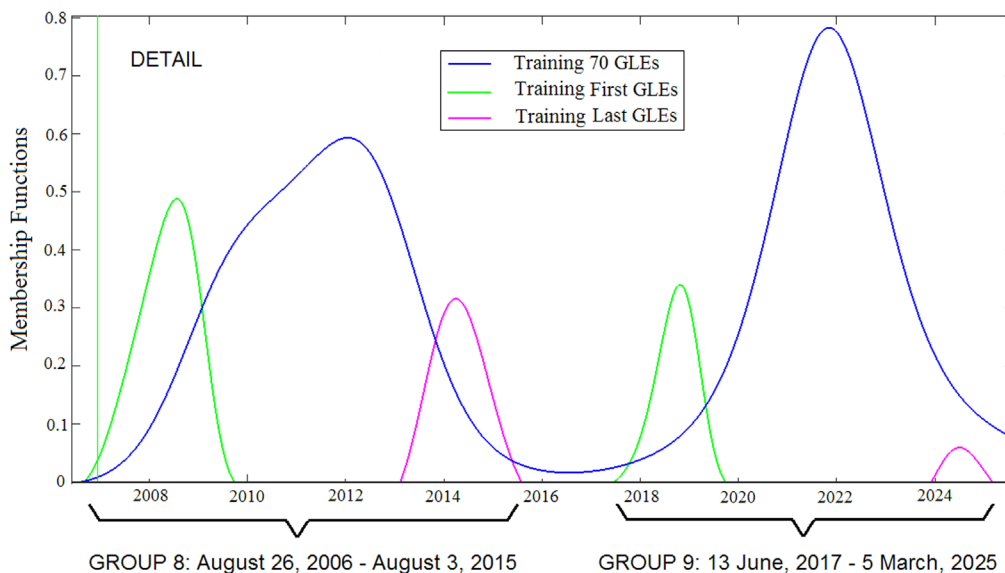


Figure 5. Detail of Groups 8 and 9 from Figure 4.

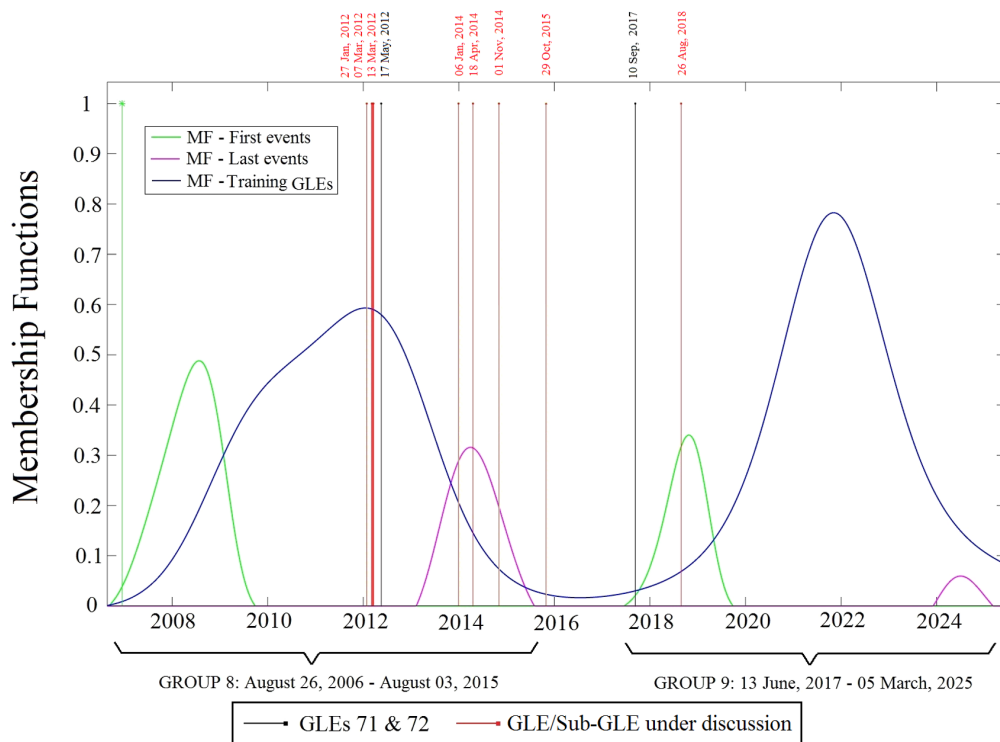


Figure 6. GLE in the prognosis zone.

agreement with previous works where the periodic nature of the GLE events had already been included as evidence, Pérez-Peraza et al. (2011).

Based on this MF, we can extend the analysis to later times after the 70 GLE of training. This is where the importance of our study lies, as it gives us a powerful tool to calculate the values of the MF associated with the possible occurrence of the events under study (Groups 8 and 9 of Figure 1). However, as we can see in Figure 2 even when groups can be defined very clearly for the 70 GLE of training, we observe that the date ranges for groups 8 and 9 are not well defined. In order to refine such a limitation, two subgroups are defined of each group established in Figure 2, see Table 2: a first GLE and a last GLE.

Using these subgroups as training groups, we obtain Figure 3.

In Figure 3 the ranges of groups 8 and 9 are extracted from the MFs of the first GLE and last GLE subgroups.

In order to better visualize the above, we combine Figures 2 and 3 in a new figure (Figure 4).

The above graph now allows us to obtain the prediction ranges that clearly define the end date of group 8 and the beginning and end of group 9, which is shown in detail of Figure 5. In Figure 6, we indicated the specific dates of the prognosis of events disclosed in Figure 5.

As we observed, cycle 24 was a particular period due to the appearance of peculiarly weak GLE as reported in different

works in the literature; this led to the discussion in the international scientific community about the occurrence or not of certain GLE events, such questions were derived in the reconceptualization due to the fact that there is at present a high discrepancy in the classification of events of cycle 24.

### 3. Discussion

In this discussion, the contribution we intend to render through our work is to provide more tools within the framework of the Wavelet technique and Fuzzy Logic in order to extend the study to cycles 24 and 25, as well as to corroborate the previous studies of the harmonic behavior of the GLE evidenced in Pérez-Peraza et al. (2011) and Pérez-Peraza & Juárez-Zuñiga (2015).

In Pérez-Peraza & Juárez-Zuñiga (2015), two indices were used, one of solar activity (SS) and another modulated by solar activity (RCG): eight periodicities or harmonics were obtained for each index, so in the process of working with Fuzzy Logic, 16 periodicities were applied. As mentioned above, it is important to note that in the present work only indices of solar activity were used (SS Index, F10.7 Index, and Flare Index) and the periodicities that best grouped the occurrence of the 70 GLE of logic training were selected through the obtained MF, Figure 1. This is of great importance since previously only the 11 yr periodicity of the sunspot index was used to try to group the occurrence of GLE. In our present study, we note that the grouping is more precise, Figure 2, since the grouping comes from the MF resulting from the selected harmonics, Figure 1 and Equation (2). This is to be taken into account as an important sample of the periodic behavior of the phenomenon of solar flares, which is broader and more precise than just considering the 11 yr period of sunspots.

Subsequently, we focused our analysis on the time zone after the 70 GLE of training, which we designate as the area of prognosis, where two groups were clearly formed. The treatment in fuzzy logic to obtain the start and end dates of groups eight and nine of prognosis, Table 2, was limited to the first GLE and last GLE of each group obtained in the training zone, Figure 3 and Table 1.

Once the time ranges of groups eight and nine have been obtained in the prognosis area, in order to corroborate the validity we used the method developed in Pérez-Peraza & Juárez-Zuñiga (2015). The relative profiles in the global MN network were reviewed to identify the events that actually indicate an increase at ground level.

From the above, we can observe that seven events, including GLE71, fall within the predicted time range for group eight; one more, the event of 2015 October 29, falls very close to the end of the same group eight predicted for 2015 August 3. Finally, we see that the two remaining events, including the GLE72, occur within the range of the first events predicted for group nine.

### 4. Conclusions

Among the important conclusions, we point out the following:

- a. Indexes specific to solar activity are used.

- b. The conjunction of Wavelet and Fuzzy Logic methods allows us to find the seven harmonics that accurately describe the periodic behavior of the phenomenon of solar flares. In so far as the 70 GLE of training, we define in a precise way seven groups of GLE and the beginning of an eighth group.
- c. The MFs obtained, extrapolated to later times, in the prognosis area, defined the completion of group eight and the entire interval of group nine.
- d. To verify the veracity of the prognosis, we contrast it with the events that occurred in cycle 24, which have been mentioned in the literature.
- e. It was found that nine events fall within the two groups of the prognosis area and one more falls very close to the end of group eight.

In summary, the above corroborates the potential of the method for the study of the periodic nature of the occurrence of GLE events.

To the Instituto de Geofísica of UNAM for his economical support and to the CONACyT for the economical support by means of a scholar grant. To the institutions where we obtained the databases of solar indices:

- a. Number of Sunspots (SS) from 1749 to 2017 (<http://www.sidc.be/silso/datafiles>).
- b. Solar Flux Index (F10.7) from 1947 to 2017 (<https://www.esrl.noaa.gov/psd/data/correlation/solar.data>).
- c. Flare Index (FI) from 1966 to 2014 ([http://www.koeri.boun.edu.tr/astronomy/fi\\_nedir.htm](http://www.koeri.boun.edu.tr/astronomy/fi_nedir.htm)).

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