UPPER CUTOFF OF HIGH ENERGY SOLAR PROTONS

Dj. HERISTCHI, G. TROTTET, and J. PEREZ-PERAZA*
Observatoire de Meudon, 92190 Meudon, France

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Abstract. By studying the data from the worldwide neutron monitor network the spectra of most of the solar proton events in cycles 19–20 have been determined. These spectra are best represented by a power law with an upper cutoff R_m. This holds over a wide range in energy or rigidity. For the events examined R_m had values between 3 GV and 20 GV. It is shown that there is no correlation between R_m and the amplitude of the events.

The equation describing continuous particle acceleration in a confining medium is solved in the non-stationary case. This solution shows the existence of a cutoff in the spectrum, and is compared with the experimental results in connection with the problem of particle acceleration time.

1. Introduction

Evidence has been given by Heristchi and Trottet (1971) for the existence of an upper cutoff in the solar proton spectrum. By upper cutoff is meant an energy or rigidity level beyond which there are no accelerated particles. The authors have shown that the best agreement with the observations, in the low (100–500 MeV) as well as in the high energy ranges, is obtained with a differential spectrum represented by a power law with an upper cutoff. They have also demonstrated that this upper cutoff remains constant for the entire duration of an event. Recently, Heristchi and Trottet (1975) have shown that a power law with an upper cutoff also agrees with the direct measurement of the spectrum from satellite observations by Vernov *et al.* (1973).

It should be noted that the upper cutoff may not be quite as sharp as is here assumed. For example, the spectrum may also be represented by a rapid change of the power law exponent at high energies, but lack of experimental data in the high energy region prevents such a detailed determination of this part of the spectrum.

The solar proton spectrum is generally represented by an exponential or power law up to an infinite energy or rigidity. Such spectra, although consistent with observations over limited energy ranges, diverge from the experimental curves for wider ranges. It should be noted that the exponent of the power law, as determined from neutron monitor (NM) data, is larger where the upper cutoff is not taken into account than when it is.

In this paper the upper cutoff of proton events recorded at ground level (GLE) during solar cycles No 19 and No 20 is determined from the records of the worldwide neutron monitors network. This cutoff provides us with a directly

* Instituto de Astronomia, Universidad Nacional Autonoma de Mexico, Mexico City, Mexico.

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measurable parameter of the accelerating source, as it is almost unaffected by the propagation of particles in the interplanetary medium. The results obtained are discussed in relation to the equations describing particle acceleration.

2. Determination of the Upper Cutoff

2.1. Methods

The methods employed consist in using the worldwide network of NM as an energy spectrometer. Mountain stations have been ignored. The percentage increase recorded by all the stations are related to a pressure of 760 mm Hg by a double correction of the barometric effect proposed by Palmeira *et al.* (1970). For this correction the attenuation lengths used for galactic and solar particles are respectively $\lambda_g = 140 \text{g cm}^{-2}$ and $\lambda_f = 103 \text{ g cm}^{-2}$ (Wilson *et al.*, 1967). The variations of these parameters from flare to flare and with latitude are neglected. This is reasonable as all the stations considered here have mean pressure around 760 mm Hg. Assuming that all the particles are protons, and introducing an upper cutoff R_m in the formula given by Palmeira *et al.* (1970) the percentage increase F for a NM may be formulated as follows:

$$F = \frac{A}{N_g} \int_{R_c}^{R_m} \left(\frac{\mathrm{d}J}{\mathrm{d}R}\right)_f S(R)\, \mathrm{d}R, \qquad (2\text{-}1)$$

where: A is a constant; N_g the counting rate, due to galactic cosmic rays, of a standard NM located in a place of geomagnetic cutoff R_c; R the rigidity of the protons; $(\mathrm{d}J/\mathrm{d}R)_f$ the differential spectrum of the solar protons; $S(R)$ the proton specific yield function (S.Y.F.). The values obtained by Carmichael *et al.* (1965) have been used for N_g and those calculated by Shea *et al.* (1968) for R_c. Use is made of the S.Y.F. given by Lockwood *et al.* (1974). In order to simplify numerical calculation this function is represented by power laws in different rigidity bands and $(\mathrm{d}J/\mathrm{d}R)_f$ is taken as proportional to $R^{-\mu}$. Under these conditions the parameters to be determined are A, μ, and R_m. An expression equivalent to (2-1) can be written as a function of energy by taking $E^{-\gamma}$ for the differential spectrum and letting E_m correspond to R_m, E_c to R_c, and $S(E)$ to $S(R)$. In this case the unknown parameters become A', γ and E_m.

Two methods are used to determine the unknown parameters.

Method 1

This method uses a large number of stations located at different geomagnetic latitudes. For each interval of time the parameters (A, μ, R_m or A', γ, E_m) are determined by applying the method of least squares. As the stations have different asymptotic direction of viewing, such a method is only applicable to the case of an isotropic event. However it remains valid in cases of small anisotropy or in the decreasing phase of an event where the isotropy is generally established.

Method 2

In this method the ratio of the counting rates of two NM stations located at different geomagnetic latitudes is used. Figure 1 gives two examples of this ratio K as a function of R_m for different values of μ. The shape of these curves shows that for a given μ, K varies rapidly when R_m is slightly greater than the larger geomagnetic cutoff, and remains practically constant as R_m increases. Then for a given K, R_m is quite independent of μ in the first region and μ quite independent of R_m in the second one. Thus, to apply the method two pairs or three stations are chosen: one in the polar region (cutoff R_{c1}), one at the lowest latitude where the amplitude of the event is measurable (cutoff R_{c3}) and another at an intermediate latitude (cutoff R_{c2}). Then R_m and μ can be determined by use of an iterative method. Indeed starting from $R_m \gg R_{c3}$, μ is obtained from $k_{12} = F_1/F_2$. This value of μ gives a new R_m when using $k_{13} = F_1/F_3$. This new R_m is used to determine a new μ etc. This process is rapidly convergent. Of course if one of the two parameters is known, the other one is easily determined.

The advantage of this method is that only a small number of stations is used. Moreover, it is applicable to anisotropic events if the chosen stations have similar mean asymptotic directions of viewing.

The upper cutoff can be estimated from the latitude effect of an event (Shea and Smart, 1973). Such a method does not lead to a good determination of R_m,

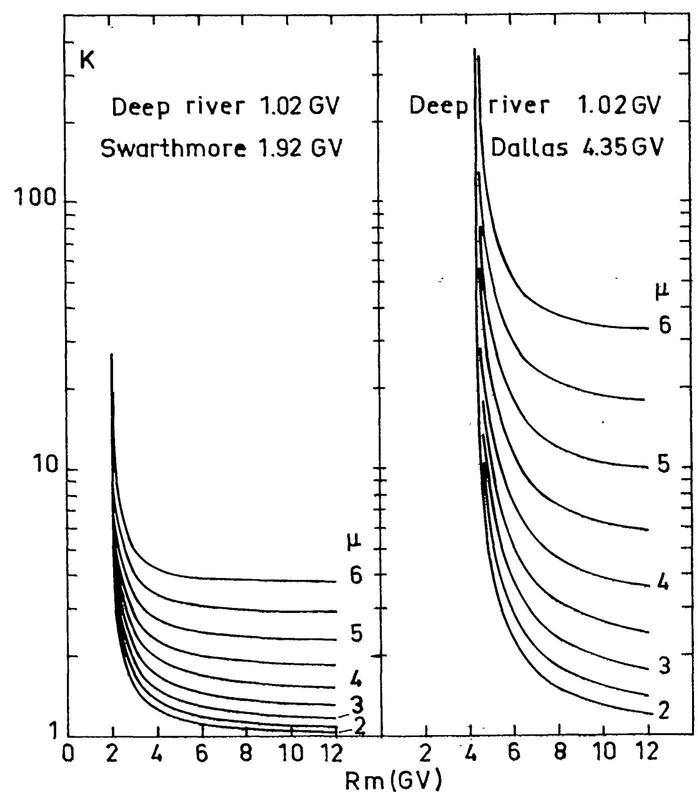

Fig. 1. Two examples of the calculated ratio k between the counting rates of two NM stations versus R_m for different values of μ.

but it can be used to verify the results obtained by the other methods. Indeed a station with a cutoff greater than R_m should not register the event, although it is possible, if the cutoff is only slightly greater than R_m, it could exhibit a small increase in level. This is mainly due to: (i) the use of the vertical cutoffs in this paper, though they may have lower values in other directions; (ii) the shape taken for the upper cutoff which may not be as sharp as we have assumed; (iii) the existence of a penumbral band.

2.2. Application of the methods

Fifteen GLE occurring through the solar cycles No. 19 and No. 20 have been investigated by means of the methods described above. The results are summarized in Table I. In addition it seems useful to discuss each event separately in connection with other observations. If E_m or R_m is smaller than the highest energy registered at the Earth (Shea and Smart, 1973) we discuss this discrepancy.

February 23, 1956 Event

Since this event is detected by low latitude NM stations, the value of R_m is large. The ratio between two stations, the cutoffs of which are R_{c1} and R_{c2} ($R_{c1} < R_{c2}$), is nearly independent of R_m if R_{c2} is low compared to it. So μ can be determined without knowing the exact value of R_m. The ratio of 15 minute-counts at Chicago against those of the *NM* aboard the Wellington Harbour ship is used. Since this ratio is constant during 2 to 3 hours from 0500 UT, it can be assumed that the anisotropy present at the beginning of the event has disappeared. Taking R_m to

TABLE I

Date	Largest sea level increase %	E_m (GeV)	R_m (GV)
February 23, 1956	4554.0 ± 12.0		$20.0^{+10.0}_{-4.0}$
May 4, 1960	290.0 ± 10.0		7.0 ± 1.0
September 3, 1960	4.5 ± 1.0		5.0 ± 2.0
November 12, 1960	135.0 ± 4.0	3.3 ± 0.6	{ 4.5 ± 0.8 3.8* ± 1.0
November 15, 1960	160.0 ± 2.0	3.1 ± 0.4	4.1 ± 0.8
November 20, 1960	8.0 ± 1.0		3.7 ± 1.0
July 18, 1961	23.5 ± 1.4	3.2 ± 0.7	4.3 ± 0.9
July 7, 1966	2.5 ± 0.3		3.2 ± 0.7
January 28, 1967	21.0 ± 1.0	4.7 ± 0.5	5.7 ± 0.7
November 18, 1968	14.0		5.7 ± 1.5
February 25, 1969	16.0		5.7 ± 0.9
March 30, 1969	8.8 ± 0.2	3.7 ± 0.6	4.5 ± 0.7
January 24, 1971	26.0 ± 1.0	3.0 ± 0.5	4.2 ± 0.6
September 1, 1971	16.0 ± 0.5	2.4 ± 0.5	3.4 ± 0.6
August 7, 1972	7.0 ± 1.0		6.6 ± 1.0

* after correction of geomagnetic cutoffs.

be larger than 20 GV the value of μ is found to be 5.6 ± 0.3. From the hourly counts of two other stations, Stockholm and Gottingen, we obtain $\mu = 5.7 \pm 0.3$. It should be noted that these values, differ from those found by McCracken (1962) and Meyer *et al.* (1956). This is certainly due to the use of a different S.Y.F.

To determine R_m, the use of an equatorial station is necessary. Since there is no such station at sea level, we have considered Huancayo, at an atmospheric depth of 680 mb. Using the sea level proton specific yield function and the differential responses given by Lockwood and Webber (1967) for sea level and 680 mb, we have determined the proton S.Y.F. at 680 mb.

From Chicago and Huancayo we obtain $R_m = (20^{+10}_{-4})$ GV. This result can only be considered as an indication as the method is not precise.

Sarabhai *et al.* (1956) consider the increases of the indian stations as due to particles from 35 to 65 GeV. They determine these energies by considering the asymptotic direction of viewing of these stations, and assuming that the particles propagate from the sun inside a solid angle of 30°. In fact this kind of determination of the energy of the particles is unsatisfactory taking into account our actual knowledge on particle propagation in interplanetary space.

May 4, 1960 Event

This event is short lived and very anisotropic (McCracken, 1962). From 15 minute-counts of Deep River and Lincoln and from 2 minute-counts of Deep River and Berkeley we obtain:

$$\mu = 3.9 \pm 0.5,$$
$$R_m = (7.0 \pm 1.0)\ \text{GV}.$$

It should be noted that the ratio between stations is roughly constant during several consecutive intervals of time.

The above value of μ which corresponds to $\gamma = 3$ is similar to that found from balloon measurements by Charakhchyan *et al.* (1962a).

September 3, 1960 Event

For a four hour period around the maximum, we obtain, from Deep River-Chicago and from Uppsala-Lindau:

$$\mu = 4.0 \pm 0.6,$$
$$R_m = (5.0 \pm 2.0)\ \text{GV}.$$

The above value of μ corresponds to $\gamma = 3$. This value agrees with the spectra determined by Charakhchyan *et al.* (1962a) (200–400 MeV) and Biswas *et al.* (1962) (300–1000 MeV) from balloon measurements.

Winckler *et al.* (1961) obtain $\gamma = 4$ in the energy range 100–400 MeV.

November 12, 1960 Event

This event occurs during a very perturbed period with respect to geomagnetic and cosmic ray phenomena. In fact a SSC started just after the onset of the event and a large geomagnetic storm ($Kp=9$) followed. In addition a Forbush decrease started some hours later.

The time profile of the event presents two maxima. According to McCracken (1962) there is anisotropy till the beginning of the second increase, for which the isotropy is established.

Because of the presence of the Forbush decrease the galactic level is difficult to determine. In order to make such a determination, we obtain the background of each station for each interval of time by using the smoothed time profile of the Forbush decrease recorded at a low latitude station which does not see the proton event. The latitude effect and the eventual anisotropy of the Forbush decrease are neglected.

The first method applied to the isotropic part of the event gives:

$$E_m = (3.3 \pm 0.6)\ \text{GeV}; \quad R_m = (4.3 \pm 0.7)\ \text{GV}.$$

γ and μ vary respectively from 0.75 and 0.9 at the beginning of the second increase to 2.6 and 4.0 in the decreasing phase.

For the same part of the event the second method applied to different combinations of stations gives:

$$R_m = (4.4 - 4.7 \pm 0.8)\ \text{GV},$$
$$\mu = 1 - 4.2.$$

However the cutoffs are influenced by the geomagnetic storm and they can decrease by a quite significant value; this makes the above results uncertain. To take this variation of the cutoffs into account we proceed as follows:

The variation ΔR of the geomagnetic cutoff as a function of the a_p index and of the geomagnetic latitude λ can be written as follows (Louis, 1972):

$$\Delta R(\text{GV}) = (1.5 \times 10^{-2}\, a_p + 1.3) \sin^2\left(\frac{\lambda}{2}\right) \tag{2-2}$$

for $30° < \lambda < 50°$ and $a_p > 39$.

As this formula is not applicable to high latitude stations we cannot determine μ. So we take $\mu = 5.5$, a value which corresponds to the spectrum determined by Ney and Stein (1962) from nuclear emulsion measurements. Then we determine R_m from a polar station, where the change of R_c has little effect on the counting rate, and Lindau, Prague, Munich and Limeil, for which the cutoffs have been corrected by using the formula [2-2]. We obtain:

$$R_m = (3.8 \pm 1.0)\ \text{GV}.$$

It should be noted that the diminution of the geomagnetic cutoff leads to an increase of the counting rate even for equatorial stations. Such an augmentation can be sometimes identified as a proton event (Roederer *et al.*, 1961). Nevertheless a station (i.e.: Jungfraujoch $R_c = 4.5$ GV) with a cutoff slightly larger than R_m may record the event because of the mentioned diminution of cutoff.

According to Shea and Smart, 1973, the highest energy registered is greater than 4.1 GeV (5 GV). This value must be lowered because of the diminution of the cutoffs. Moreover Hermanus ($R_c = 4.9$ GV) presents an increase (see Carmichael and Steljes, 1961) but this occurs several hours before the other stations.

November 15, 1960 Event

About one hour after its onset this event becomes isotropic. Method 1 applied to bihourly data gives $E_m = (3.1 \pm 0.4)$ GeV, $\gamma = 3.1 \pm 0.3$; $R_m = (4.1 \pm 0.7)$ GV and $\mu = 4.0 \pm 0.4$. It can be seen on Figure 2 that the values of E_m and R_m remain quite constant during several intervals of time. On Figure 3, the latitude effect of

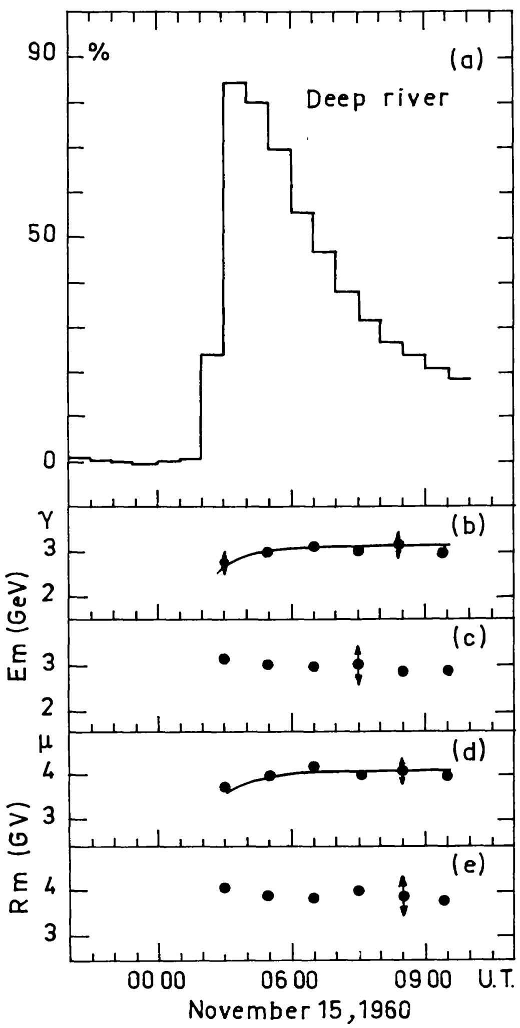

Fig. 2. Time behavior and spectral parameters of the solar proton event of 15 November 1960. (a) Recordings by a typical station. (b) Exponent of the differential energy spectrum. The solid curve is calculated from Krimigis's diffusion model (1965). (c) Upper cutoff in the differential energy spectrum. (d), (e) same as (b) and (c) for rigidity.

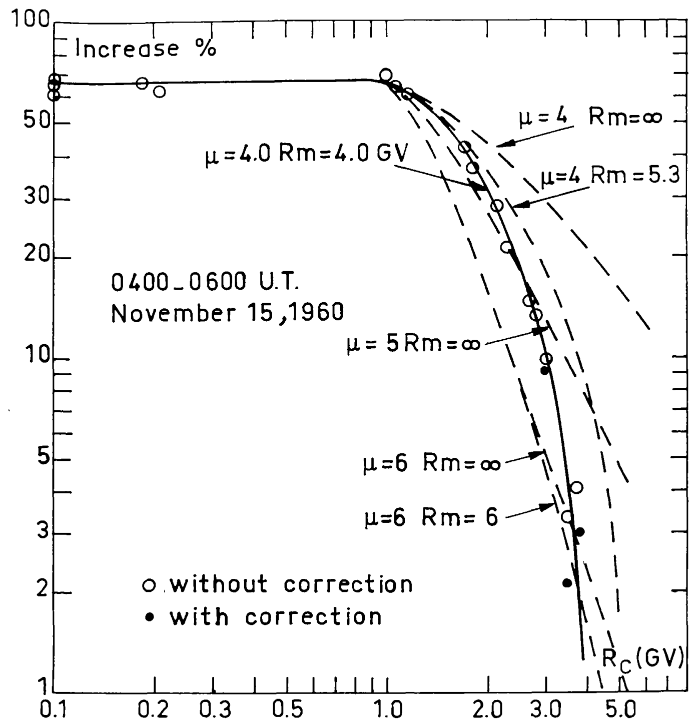

Fig. 3. Percent increase of the sea level neutron monitors compared with different theoretical curves. Cosmic ray level correction is made by using a low latitude station.

the increase and the corresponding curve calculated with the above values of R_m and μ are plotted against rigidity. This figure shows that, on one hand the experimental points agree with the expected curve and that, on the other hand, a spectrum which does not take R_m into account cannot explain the observations. The second method applied to several combinations of stations gives a mean reading of $R_m = (4.2 \pm 1.0)$ GV and $\mu = 4.0 \pm 0.6$. The value of μ agrees with the spectra measured by Ney and Stein (1962) and Charakhchyan *et al.* (1962b) who give $\gamma = 2.8$. Lockwood and Shea (1961) take $R_m = 6$ GV and find $\mu = 6$. From Figure 3 it is clear that such a spectrum cannot agree with the latitude effect of the event.

The detailed records of the Jungfraujoch, the vertical cutoff of which is 4.5 GV, show that the event has been clearly detected by this station. Considering its high altitude and the earlier remarks concerning the use of the vertical values for the geomagnetic cutoffs, this is not incompatible with the value given here for R_m.

The galactic level presents rather large variations during the period surrounding the event and some stations with cutoffs substantially larger than R_m detect an increase which by chance coincides with the event but which may not be associated with it. Indeed if R_m is taken as 5.3 GV (cutoff of Pic du Midi) with $\mu = 4$ corresponding to direct measurements, the concordance between the experimental points and the calculated latitude effect is destroyed (see Figure 3) and most of the stations should present a much larger increase.

November 20, 1960 Event

This event is isotropic and of small amplitude. It occurs during a period of large diurnal and other short-term fluctuations of the galactic background (Carmichael and Steljes, 1962). The results of method 2 applied to Uppsala, Leeds, Lindau, are:

$$R_m = (3.7 \pm 1.0)\ \mathrm{GV} \quad \text{and} \quad \mu = 2.4 \pm 0.6.$$

July 18, 1961 Event

From method 1 we obtain:

$$E_m = (3.2 \pm 0.7)\ \mathrm{GeV}, \quad \gamma = 3.4 \pm 0.5, \quad R_m = (4.3 \pm 0.9)\ \mathrm{GV}, \quad \text{and} \quad \mu = 5.0 \pm 0.6.$$

Method 2 applied to two groups of 3 stations gives similar results.

This event occurs during a period of both large fluctuations in the galactic background and during a geomagnetic storm. Thus it is difficult to determine the galactic level. This difficulty is increased for the European stations which have been used to determine R_m, because they record an increase of a few percent some hours before the onset of the event. In such conditions the above results only give an indication of the values of the parameters. The value of γ is smaller than that deduced from balloon measurements (Hofmann and Winckler, 1963).

July 7, 1966 Event

The determination of R_m is possible only if μ is known, because this event is anisotropic (Carmichael, 1969) and of very small amplitude. Then, using $\mu = 4.5$ (corresponding to $\gamma = 3.4$) as determined by Heristchi *et al.* (1969), R_m is found to be (3.2 ± 0.7) GV. This value is obtained from the ratio of the 15 minute records, of Kerguelen-Kiel on one hand and of Kerguelen-Leeds on the other hand, for one hour centred on the maximum. The small amplitude makes the value of R_m rather uncertain but as the event is clearly registered by Leeds, it can be assumed that R_m is larger than 2.2 GV.

January 28, 1967 Event

A detailed study of this event in which the evolution with time of E_m, γ, R_m and μ are examined, has been presented by Heristchi and Trottet (1971). With the revised S.Y.F., method 1 gives: $E_m = (4.7 \pm 0.5)$ GeV, $\gamma = 3.3 \pm 0.2$, $R_m = (5.7 \pm 0.7)$ GV and $\mu = 4.5 \pm 0.2$. Method 2 has been applied here to two groups of 3 stations, in the European and the American sectors. From Kerguelen, Leeds, Lindau, we obtain: $R_m = (5.6 \pm 0.7)$ GV and $\mu = 4.4 \pm 0.3$; from Deep River, Swarthmore, Dallas: $R_m = (5.8 \pm 0.7)$ GV and $\mu = 4.6 \pm 0.3$. These values corroborate those obtained from method 1. A comparison, between these results and those obtained with the S.Y.F. given by Lockwood and Webber (1967), shows that the differences between the values of the two sets of parameters are small, the present ones being slightly larger.

The value of γ found here agrees with that measured by Barcus (1969) from balloon experiments but it is smaller than that found by Ageshin *et al.* (1969).

November 18, 1968 Event

This event is very anisotropic (Duggal *et al.*, 1971; Tanskanen, 1970) and short lived. It has been mainly registered by the stations in the American sector. Method 2 has been applied to the 15 minute records of Deep River, Durham and Dallas. During three intervals of time around the maximum we obtain: $R_m = (5.7 \pm 1.5)$ GV and $\mu = 4.4 \pm 1.0$.

February 25, 1969 Event

This event is exceptionally anisotropic (Duggal and Pomerantz, 1971) and it is mainly detected by the American stations. In this sector the choice of mid and low latitude stations is limited to Swarthmore and Dallas. In order to find which polar station to include we have calculated the mean asymptotic directions of viewing of several stations by using Shea's *et al.* (1968) method. In this computation the proton spectrum is taken as R^{-4} with $R_m = 6$ GV. The results show that the best choice of stations is Deep River and Goose Bay though their mean asymptotic longitudes are slightly different from those of Swarthmore and Dallas. The results of method 2 are: $R_m = (5.7 \pm 0.9)$ GV and $\mu = 4.4 \pm 0.7$. This value of μ agrees with $\mu = 4.5$ deduced from balloon measurements by Bazilevskaya *et al.* (1971 a and b) and is compatible with $\mu = 3.9$ corresponding to $\gamma = 2.7$ obtained by Barouch *et al.* (1970).

March 30, 1969 Event

Method 1 has been applied to hourly data. The values obtained for the parameters are: $E_m = (3.7 \pm 0.6)$ GeV, $\gamma = 0.9 - 1.8$, $R_m = (4.5 \pm 0.7)$ GV and $\mu = 1 - 2.2$. The time behaviour of these parameters has been presented by Perez-Peraza (1972). Method 2 applied to several groups of 3 stations gives similar values of R_m and μ.

Bukata *et al.* (1970) remark that there is a small anisotropy at the time of the maximum and that the increase of 1% observed at Pic du Midi is due to the event. In fact we think that these two effects are due to the diurnal variation visible with an amplitude of about 1%, at Rome, Alma Ata, Chacaltaya. Indeed the phase of the diurnal variation observed from these stations corresponds to the time of the increase at Pic du Midi provided that difference between the mean asymptotic longitudes is taken into account. When the galactic background is corrected for the diurnal variation from the smoothed records of Rome, the values found for E_m and R_m remain the same with or without correction and $\gamma = 1.7 - 2.2$, $\mu = 1.9 - 2.8$.

Balloon measurements by Bazilevskaya *et al.* (1971a, b) give $\gamma = 2.0 - 2.4$ from 1000 UT to 1800 UT. This is compatible with our results.

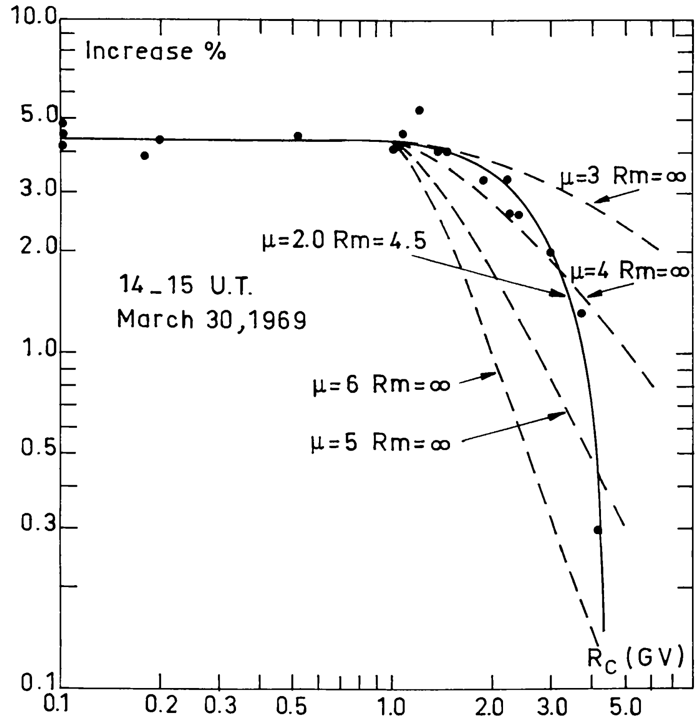

Fig. 4. Percent increase of the sea level neutron monitors compared with different theoretical curves.

The percent increase without background correction of the different stations is plotted on Figure 4 as a function of rigidity.

January 24–25, 1971 Event

This event has already been discussed by Heristchi *et al.* (1972). With the revised S.Y.F., E_m and R_m remain, here again, constant in time within the errors. Their mean values are: $E_m = (3.0 \pm 0.5)$ GeV and $R_m = (4.2 \pm 0.6)$ GV. We obtain: $\gamma = 3.0 - 3.2 \pm 0.4$, $\mu = 3.8 - 4.6 \pm 0.6$. Figure 5 shows the latitude effect for this event.

September 1–2, 1971 Event

As in the preceding case, this event has been studied by Heristchi *et al.* (1972). With the revised S.Y.F. we obtain $E_m = (2.4 \pm 0.5)$ GeV, $\gamma = 3.0 \pm 0.4$, $R_m = (3.4 \pm 0.6)$ GV and $\mu = 4.1 \pm 0.4$.

Figure 6 shows the latitude effect of the event. Lockwood *et al.* (1974), who do not take R_m into account, find $\mu = 5.5$. The latitude effect given by these authors does not indicate the increases recorded by most of the mean latitude stations. Moreover their determination of μ is based upon an increase of 0.5% at Dallas. On the other hand Shea and Smart (1973) give 3.6 GeV for the highest energy registered. From five minute as well as hourly data, compiled by Coffey (1972), we do not find any increase in the records of Dallas during the event.

A differential spectrum, having the form of a power law with an upper cutoff, has been applied by Heristchi and Trottet (1975) to the satellite observations of

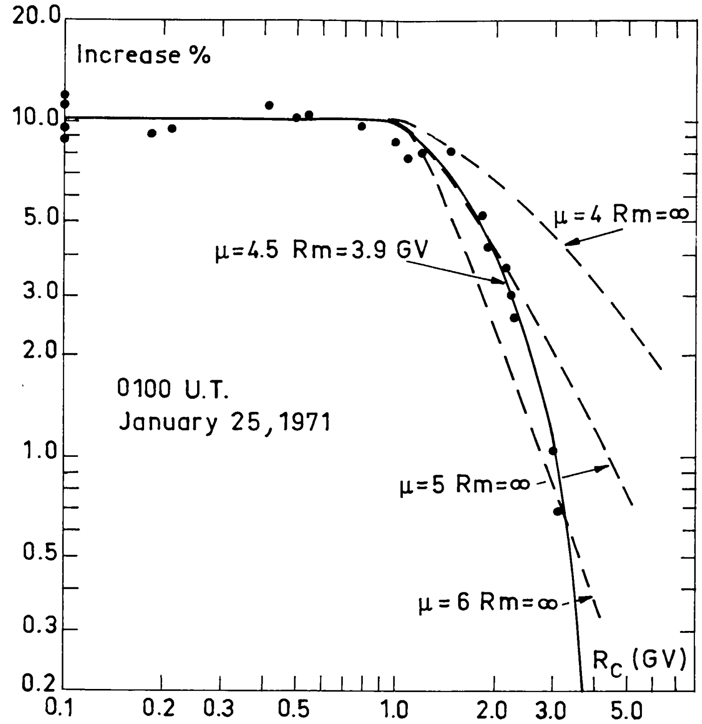

Fig. 5. Percent increase of the sea level neutron monitors compared with different theoretical curves.

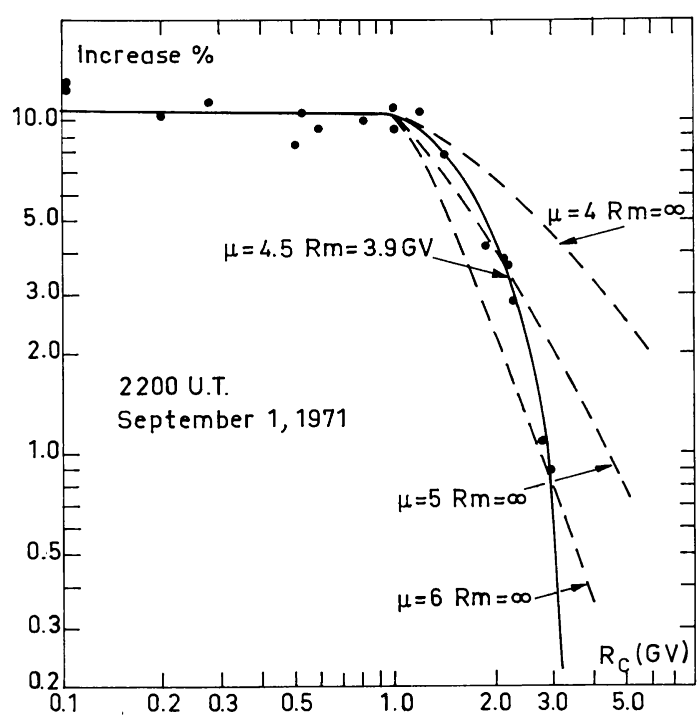

Fig. 6. Percent increase of the sea level neutron monitors compared with different theoretical curves.

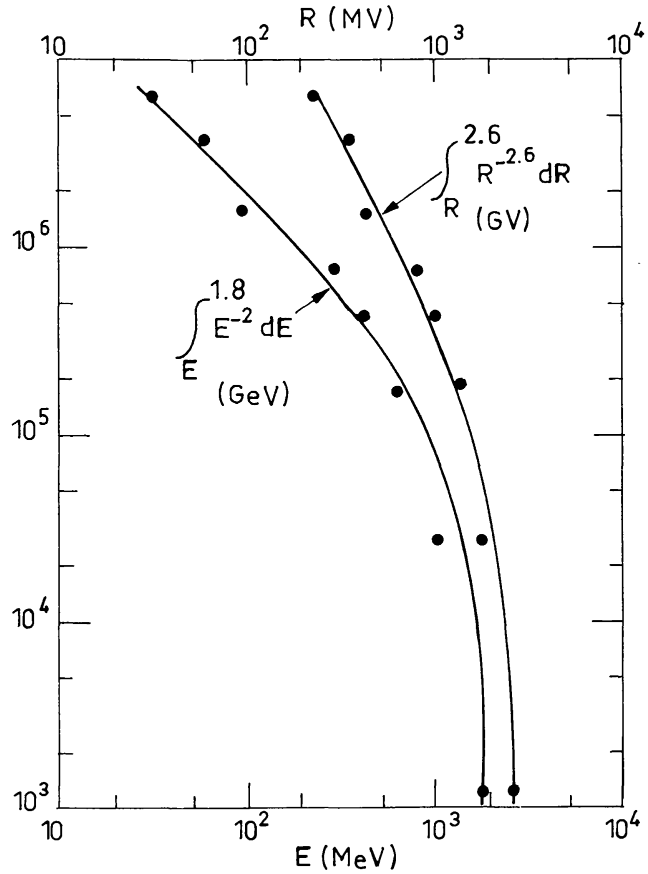

Fig. 7. Integral spectrum of the 1–2 September 1971 proton event. Experimental points are from Vernov *et al.*

Vernov *et al.* (1973) and the parameters obtained are: $E_m = (1.8 \pm 0.5)$ GeV, $\gamma = 2.0 \pm 0.3$, $R_m = (2.6 \pm 0.6)$ GV, $\mu = 2.6 \pm 0.4$.

The difference between these latter values and those obtained from NM data is discussed by Heristchi and Trottet (1975). Figure 7 shows that a power law with an upper cutoff agrees with Vernov's observations.

August 7, 1972 Event

As the galactic level was variable during the event, we have applied background corrections which are different for the American and the European sectors. Method 2 applied to an American group of 3 stations and to an European one gives in the two cases: $R_m = (6.6 \pm 1.0)$ GV and $\mu = 4.7 \pm 0.7$. This corresponds to $\gamma = 3.5$. This value is comparable to $\gamma = 4$ given by Bazilevskaya *et al.* (1973) from balloon measurements on August 8.

It is to be noted that Swinson (1973), who noticed that the event has been registered by the Bolivian underground meson telescope, proposes an upper limiting rigidity of 20 to 25 GV. In fact, an increase is visible on the records of this detector, but it starts before the onset of the responsible flare. Moreover it would be surprising that such a station should register the event, while NM stations, even mountain ones, with larger cutoffs than R_m, do not present any increase (Chasson, 1973).

3. Discussion

3.1. Evidence for the existence of an upper cutoff

The arguments proposed to demonstrate the existence of an upper cutoff in the spectrum of the January 28, 1967 event are summarized in the introduction of this paper. The same arguments are applicable to other events.

The curves presented in Figures 3–6 show that the observed latitude effect of an event cuts across the latitude effects expected for a power law spectrum without upper cutoff. This is observed for several hours, in rigidity and in energy, for any event for which the latitude effect can be obtained. We see four possible explanations of this discrepancy: (1) the determination of the increase is uncertain; (2) the S.Y.F. is inexact; (3) the shape of the spectrum is not a power law; (4) there is an upper cutoff in the spectrum.

(1) The increases of low latitude stations, often smaller than 1%, are determined with large uncertainties, but as the rapid fall of the latitude effect at high rigidities recurs for each event, it cannot be due to this lack of precision. Nevertheless, if the observations of low latitude stations are ignored, it is possible to find γ (or μ) with E_m (or R_m) $= \infty$ leading to a latitude effect compatible with the remaining observations. For instance, in the case of the November 15, 1960 event, $\mu = 4.8$ gives a reasonable fit with the observations up to approximately 2 GV (Figure 3). Such a spectrum, on the other hand, predicts sea level increases of 10% around 5 GV and of the order of 1% around 10 GV, and these increases should be observable for several hours. In fact, such augmentations, which would be markedly larger than the uncertainties of the measurements, are not at all observed. The same discussion remains valid for the March 30, 1969 event for which $\mu = 4$, as given by Lockwood *et al.* (1974), agrees with the observations in the low rigidity range (Figure 4), but implies an increase of 1% at Dallas, which was not observed. If, on the other hand, as is sometimes done, γ and μ are determined only from the observations of high and low latitude stations, larger values are found than in the preceding case, and the latitude effect obtained does not fit the observations of mid-latitude stations. (see Figures 3–6). The above discussion, illustrated by particular examples, may be applied to all events for which the latitude effect is obtainable.

(2) The same discrepancy is observed in using the S.Y.F.'s given by Webber (1962), Lockwood and Webber (1967) and Lockwood *et al.* (1974), obtained from independent measurements. Moreover the S.Y.F. determined by Lockwood and Webber (1967) from the latitude effect of galactic cosmic rays, and that given by Lockwood *et al.* (1974) give shapes which agree well above 2 GV. It is to be noted that the latter function, which has been used in this work, furnishes the S.Y.F. in the low rigidity range. Moreover, the change of the S.Y.F. has little influence on

R_m and μ. For example, in the case of the January 28, 1967 event, with the above S.Y.F.'s we have obtained respectively: $R_m(\mathrm{GV}) = 5.4 \pm 0.6$, 5.3 ± 0.5 and 5.7 ± 0.7 and $\mu = 4.6 \pm 0.2$, 4.1 ± 0.2 and 4.5 ± 0.2. Then a power law without R_m will lead to good agreement with the observed latitude effect only if the S.Y.F. is substantially changed. Such a change may consist in lowering the S.Y.F. either below or above approximately 2 GV. In the first case there will be problems in connecting the spectrum with that of the low rigidity range (Lockwood *et al.*, 1974) and the μ found will be larger than that determined with the assumed S.Y.F. and consequently will differ more from the μ measured from balloon observations. In the second case the change would imply an uncertainty in the S.Y.F., in the domain where its experimental determination is in fact most precise. In any case, a significant change of the S.Y.F. would imply that most of the measurements used for its determination are inexact. This is not acceptable. The above considerations show then that a large inexactitude of the S.Y.F. is an untenable hypothesis.

(3) In the case of the January 28, 1967 event, Lockwood (1968) and Lockwood *et al.* (1974) show that an exponential spectrum agrees reasonably well with the observed latitude effect. This could be also true for other events. An exponential law gives such a fit because it decreases more rapidly than $R^{-\mu}$ the difference between the two curves increasing with R and leading to the same effect as $R^{-\mu}$ with R_m. However, Heristchi and Trottet (1971), have shown that an exponential spectrum extrapolated below 500 MeV does not agree at all with balloon measurements. This is also observed for other events. Thus a power law spectrum agreeing with balloon measurements (>100 MeV) must change its shape above 500 MeV (~1 GV) this change just coinciding with the change of detector type! Moreover, some direct measurements (Biswas *et al.*, 1962; Ney and Stein, 1962; Barouch *et al.*, 1970; Heristchi *et al.*, 1971) do not show any change from a power law spectrum, up to 1 GeV (~2 GV).

For higher energies there are no available direct measurements. Therefore, in order to fit the observed latitude effect, it is possible to choose different shapes for the spectrum. The rapid fall of the latitude effect in its high energy or rigidity part indicates that the spectrum must also fall very rapidly. As this spectrum must also lead to a latitude effect close to that deduced from our spectrum, the adopted shape between 2 GV and R_m cannot be far from the shape considered in this paper.

(4) When the spectrum is taken as a power law with an upper cutoff, all the difficulties disappear. In fact a good concordance with the observed latitude effect is obtained by using the available S.Y.F. and the exponents γ or μ are in good concordance with those measured from balloons.

The hypothesis of the existence of an upper cutoff is not only supported by the fact that the introduction of such a parameter leads to a good agreement between the calculated and observed latitude effects. Effectively, as it is noted by Lockwood *et al.* (1974) it is normal to get a better fit when an additional free parameter is considered. In fact the physical validity of E_m or R_m is implied by its constancy with time for each studied event. Moreover, as will be seen below, the existence of such a cutoff is a natural consequence of the particle acceleration processes. We repeat here again, that this cutoff is certainly not as sharp as we have assumed, but it is more probably a narrow domain of energy or rigidity where the spectrum falls very rapidly to zero.

The exponents γ or μ, we have found, vary slowly and regularly with time, following the variations expected from the diffusion model by Krimigis (1965) (see Figure 2). As we have already mentioned, there is a good concordance between γ or μ and those measured on balloon above 100 MeV. To make this comparison we sometimes change γ into μ. There are two ways to do this: (i) the balloon measured spectrum is extrapolated to the NM domain where its equivalent in rigidity is taken; (ii) the balloon measured spectrum is directly expressed in rigidity and then extrapolated to the NM domain. It should be noted that, although the differences in the results obtained by these two methods are often smaller than the experimental uncertainties, the first one, which implies an $E^{-\gamma}$ spectrum, leads to a slightly better agreement.

For several of the six events treated in this paper, and by Lockwood *et al.* (1974), the μ determined by us corresponds to that deduced from the satellite observations (<400 MV) presented by these authors. In fact the comparison between the μ found in the low and high rigidity ranges must be done carefully. Indeed, it is not yet established that at the accelerating source the spectrum is a power law in the whole domain of energy or rigidity. Moreover, as the propagation effects are not the same for low and high rigidities, it is difficult to know at what instant the spectrum in the low rigidities may be compared with that in the high rigidities. On the other hand, we agree with other authors in considering that $E^{-\gamma}$ or $R^{-\mu}$ leads to a good representation of the spectrum in limited domains of energy or rigidity.

In the NM domain, the comparison between our results and those of other authors who have not taken R_m into account is generally not valid because the S.Y.F.'s used are different. Nevertheless it should be noted that the μ are always smaller when R_m is taken into account. This is verified for all the events studied, here and by Lockwood *et al.* (1974) with the same S.Y.F. Even if, for some cases, this difference is not statistically significant, as it is a mathematical consequence of the introduction of R_m in the spectrum, it is normal that it exists. This alone does not provide a sufficient argument for the existence of R_m; further evidence is provided by both the agreement of our spectrum with that measured from balloons, and the agreement with the observed latitude effect.

The existence of R_m implies that a station for which the cutoff is greater than R_m cannot observe the event. We have already discussed the cases for which stations having cutoff slightly larger than R_m do observe the event. It is worth noting that sometimes in the literature the fluctuations of low latitude stations during an event are identified with the event and consequently used to determine the spectrum! We have not taken such fluctuations into account. Only the stations which clearly detect an event are considered here. For a station with a cutoff slightly smaller than R_m, the increase is determined with great uncertainty. Such uncertainty has little influence on the determination of R_m which is done mainly from the totality of the points available. This remains true when method 2 is used. Indeed Figure 1 shows that the value of R_m is not substantially changed even for a large variation of the ratio of a polar station to a station with R_c close to R_m.

In conclusion, when a power law spectrum with an upper cutoff is considered, a good agreement with the latitude effect observed at ground level is obtained, γ and μ correspond to those measured on balloons, and vary slowly with time in following the predictions of one diffusion model and E_m and R_m remain constant in time. Moreover in the case of the September 1–2, 1971 event, we have shown (Heristchi and Trottet, 1975) that such a spectrum gives a good fit with direct measurements performed on satellites.

For the events studied with method 2 the above arguments remain valid, except the one concerning the latitude effect. Moreover as this method converges, leading to finite values of R_m, this gives a further argument for its existence. Finally, the existence of an upper cutoff, equivalent to a region where the spectrum falls rapidly to zero, is to be expected theoretically as we shall discuss below. All together, the above arguments demonstrate the existence of an upper cutoff in the solar proton spectrum.

3.2. Examination of the results

Table I summarizes the results presented in Section 2.2.

E_m is given only for the events to which method 1 has been applied. However it can be seen that the relationship between E_m and R_m, which have been determined separately, corresponds to protons. Thus for all the events the upper cutoff can be given in energy by taking the equivalent of R_m. The second column shows the largest amplitude observed at sea level (from Shea and Smart, 1973). These values are only given as an indication because the data are not always obtained over the same time interval.

Examination of Table I shows that R_m varies from one event to another and that, except for the February 23, 1956 event, its range of variation extends from 3 to 7 GV, the lower limit being imposed by the type of detectors used and the class of events studied. Moreover R_m does not always vary in the same way as the amplitude of the event. This amplitude can be considered as proportional to the number of particles in the solar source but it depends also upon the location of the responsible flare (Burlaga, 1967). Thus the comparison between the amplitude and R_m cannot be made for all the events, but only for events issuing from flares having similar locations. For instance the R_m's of the November 15, 1960 and the January 24, 1971 events are roughly the same but the amplitude of the first is several times that of the second. The positions of the corresponding flares on the Sun are respectively 32°W and 50°W which slightly enhances the amplitude of the second event (Burlaga, 1967). Similar remarks can be made by comparing November 12, 1960 and February 25, 1969 events.

The above considerations demonstrate that R_m does not vary in the same way as the number of particles accelerated in the solar source.

As we have already mentioned R_m remains constant for several hours during an event (Figure 2). This means that, at rigidities of a few GV, no important deceleration is observed for the particles during their propagation in interplanetary space.

3.3. Theoretical evidence for the existence of an upper cutoff

In this section we are going to show that the existence of a cutoff in the solar proton spectrum is theoretically predicted.

A review of the different processes of acceleration is beyond the scope of this paper. Nevertheless in most of the cases, the balance of the number of particles N of one kind between the time t and $t+\mathrm{d}t$, in the energy or rigidity range E and $E+\mathrm{d}E$ per unit of energy or rigidity, is described by the following equation (Ginzburg and Syrovatskii, 1964, p. 296):

$$\frac{\partial N(E,t)}{\partial t}+\frac{\partial}{\partial E}[N(E,t)\alpha(E,t)]+\frac{N(E,t)}{T(E,t)}=q(E,t). \tag{3-1}$$

The diffusion of the particles through the medium is neglected. $q(E, t)$ is a source function which is zero when t is negative, the origin of time being chosen at the beginning of the acceleration. α is the energy gain per unit of time and T the characteristic time of confinement. As we have no knowledge of the time variations of α and T, these two parameters will be taken as independent of time to simplify the analytical treatment.

Equation (3-1) is generally solved in the stationary case. The differential spectrum thus obtained, assuming T independent of energy, is an exponential law if α is a constant, or a power law if α is proportional to the energy. The general solution can be easily obtained by taking the Laplace transform of (3-1) and by solving the obtained equation with $N(E, 0)=0$. The inverse Laplace transform of the obtained solution gives

$$N(E,t)=\frac{1}{\alpha(E)}\int_0^E \mathrm{d}E''\exp\left[-\int_{E''}^{E}\frac{\mathrm{d}E'}{\alpha(E')T(E')}\right]q(E'',t-\tau) \tag{3-2}$$

with

$$\tau=\int_{E''}^{E}\frac{\mathrm{d}E'}{\alpha(E')}.$$

In order to simplify the following discussion we will consider q constant in time and covering an energy range from E_{01} to E_{02}. For a given t, let E_{m1} be the value of E up to which $t-\tau$ is positive for any value of E'' and E_{m2} be the energy beyond which $t-\tau$ is negative for any value of E''. E_{m1} and E_{m2} are defined by

$$t-\int_{E0i}^{Emi}\frac{dE'}{\alpha(E')}=0.$$

Thus Equation (3-2) indicates the existence of three regions:

(a) When E is smaller than E_{m1}, $N(E, t)$ is independent of time and the integral is taken over the whole energy range covered by the source. The solution is the same as that obtained in the stationary case. The region $E<E_{m1}$ will be called region 1.

(b) When E is between E_{m1} and E_{m2}, $t-\tau$ has positive and negative values depending upon the value of E''. Here the spectrum is a function of time. The integral must be taken for the positive values of $t-\tau$ e.g. from an energy ε defined by

$$t-\int_{\varepsilon}^{E}\frac{dE'}{\alpha(E')}=0.$$

This non-stationary solution describes region 2. Of course E_{m1} and E_{m2} increase with time.

(c) When E is greater than E_{m2}, $N(E, t)=0$ because $q=0$ for $t<0$. This is region 3.

If q is not limited in energy and varies in time, the existence of E_{m1} remains valid, but E_{m2} becomes infinite and in region 1 the spectrum is not stationary.

These properties of (3-2) will be illustrated in some simple situations. For this we consider $T(E)=T$ independent of energy and two different source functions:

(S_1) $q(E)=Q$ when $E_{01}\leqslant E\leqslant E_{02}$ and 0 elsewhere.

(S_2) $q(E)=Q\exp(-E/E_{Th})$ when $E\geqslant E_{01}$.

With these conditions we will apply expression (3-2) to simple cases.

(a) $\alpha=a=$ constant:

In the case of the source S_1, the solutions in region 1 (for $E>E_{02}$) and 2 are respectively:

$$\begin{aligned}N_1&=QT\exp(-E/aT)[\exp(E_{02}/aT)-\exp(E_{01}/aT)],\\N_2&=QT\exp(-E/aT)[\exp(E_{02}/aT)-\exp((E-at)/aT)].\end{aligned}\tag{3-3}$$

The expressions of E_{m1} and E_{m2} are:

$$\begin{aligned}E_{m1}&=E_{01}+at,\\E_{m2}&=E_{02}+at.\end{aligned}\tag{3-4}$$

These last formulas show that $E_{m2}-E_{m1}=E_{02}-E_{01}$. Then the width of region 2 is the same as that of the source. Consequently, if the source is not too wide the

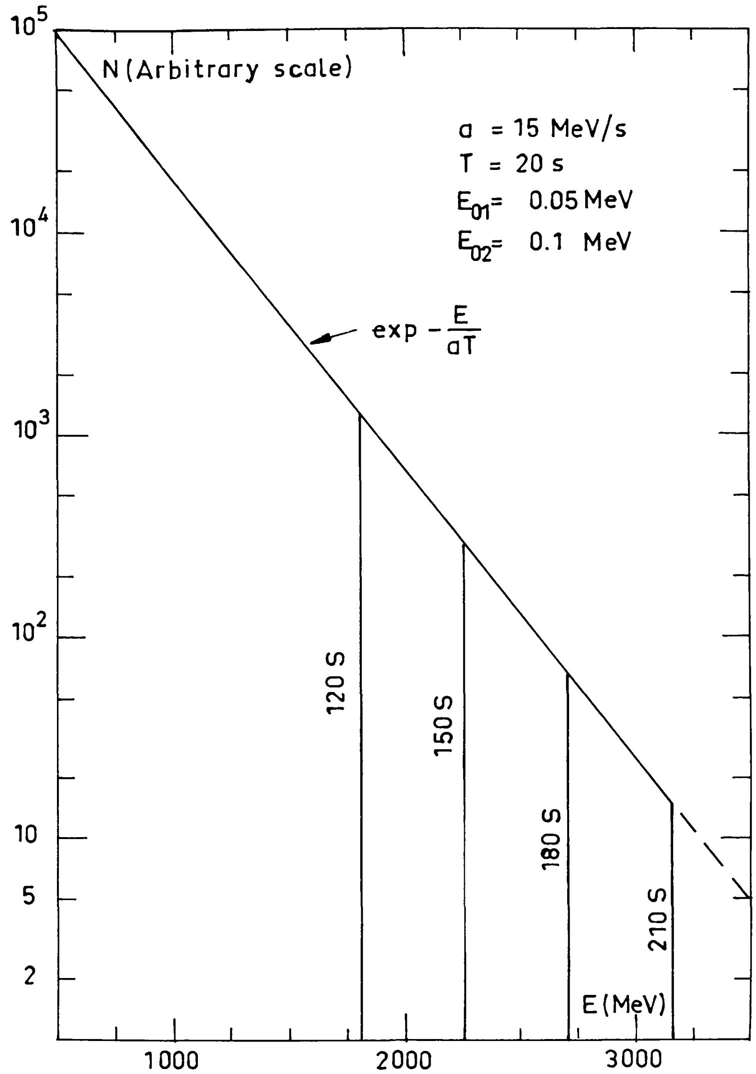

Fig. 8. Differential spectrum in the case $\alpha = a$ (see text) at different times during the acceleration.

number of particles drops rapidly to zero and there is an upper cutoff in the spectrum. This is illustrated by Figure 8 which represents N at different times for the indicated values of the parameters.

In the case of source S_2 the integration of (3-2) is easily done. Here again $E_{m1} = E_{01} + at$ but E_{m2} becomes infinite. However, for $E_{th} \ll E_{m1}$, a sharp upper cutoff is still present in the spectrum. For instance, with $E_{th} = 10\,\mathrm{keV}$, the shape of the spectrum is quite the same as in Figure 8 if we keep the same values for the other parameters.

(b) $\alpha = bE$ with $b = \text{constant}$:

With the source function S_1, in region 1, the spectrum is a power law the exponent of which is

$$\gamma = \frac{1}{bT} + 1. \tag{3-5}$$

The expressions of E_{m1} and E_{m2} are:

$$E_{m1} = E_{01} \exp(bt),$$
$$E_{m2} = E_{02} \exp(bt). \tag{3-6}$$

From Equations (3-6) we obtain $E_{m2}/E_{m1} = E_{02}/E_{01}$. This shows that the width of region 2 may be important. An example of the spectrum obtained in this case is shown in Figure 9. Here again it is clear that in region 2 the spectrum is far from the power law of region 1.

In the case of the source function S_2, the integration can be easily performed if γ is an integer. Figure 9 shows that the spectrum diverges from the power law

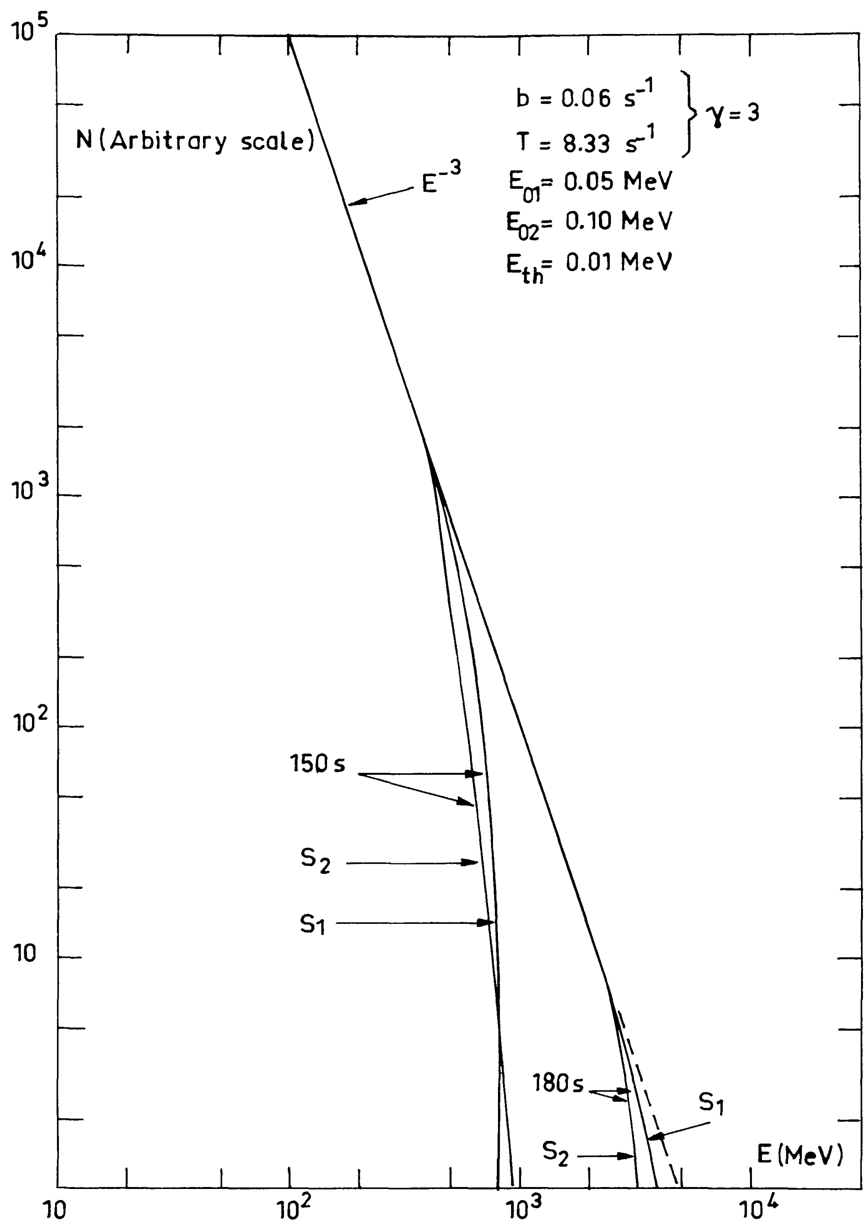

Fig. 9. Same as Figure 8 in the case $\alpha = bE$ for two source functions S_1 and S_2 (see text).

more rapidly than in the case of the source function S_1 in the low energy part of region 2.

(c) $$\alpha = \begin{cases} a = \text{constant for } E \leqslant E_1 \\ bE \ (b \text{ constant}) \text{ for } E \geqslant E_1 \end{cases} \quad \text{with } a = bE_1:$$

Here again with the source function S_1 and for $E > E_1$ we obtain a power law in region 1.

The limits of region 2 are given by:

$$\begin{aligned} E_{m1} &= E_1 \exp [b(t - (E_1 - E_{01})/a)], \\ E_{m2} &= E_1 \exp [b(t - (E_1 - E_{02})/a)]. \end{aligned} \tag{3-7}$$

These last equations give: $E_{m2}/E_{m1} = \exp [(E_{02} - E_{01})/E_1]$. Assuming that E_1 is largely outside the energy range covered by the source we have from [3-7]:

$$E_m = E_{m2} \simeq E_{m1} \simeq (E_1/e) \exp (bt);$$

under these conditions the upper cutoff is very sharp.

Similar results are obtained with the source function S_2.

All the cases examined above have shown the existence of a region where the spectrum presents a cutoff. Except for $\alpha = bE$ (in the whole energy range), this cutoff is sharp. So the existence of the upper cutoff discussed in the preceding sections is evident in the case of a continuous process of acceleration.

Up to this point we have considered a simple stage of acceleration. However the spectrum obtained at the final stage of this step can be accelerated again. Then Equation (3-1) must be solved without source function ($q = 0$) and with $N(E, 0)$ representing the available spectrum. In these conditions the solution is

$$\begin{aligned} N(E, t) &= \frac{1}{\alpha(E)} \int_0^E \mathrm{d}E'' N(E'', 0) \times \\ &\quad \times \exp \left(-\frac{1}{T} \int_{E''}^{E} \frac{\mathrm{d}E'}{\alpha(E')} \right) \delta \left(t - \int_{E''}^{E} \frac{\mathrm{d}E'}{\alpha(E')} \right) = \\ &= \frac{\alpha(E'')}{\alpha(E)} N(E'', 0) \exp \left(-\frac{t}{T} \right), \end{aligned} \tag{3-8}$$

E'' being given by $t - \int_{E''}^{E} \mathrm{d}E'/\alpha(E') = 0$. δ is the Dirac's function.

We obtain for instance:

With $\alpha = a$:

$$N(E, t) = N(E - at, 0) \exp \left(-\frac{t}{T} \right),$$

and with $\alpha = bE$:

$$N(E, t) = N(Ee^{-bt}, 0) \exp \left[-t \left(\frac{1}{T} + b \right) \right].$$

It is clear that for $\alpha = a$ region 2 has the same width as in the case of a single acceleration stage but that this width grows when $\alpha = bE$. Nevertheless the upper cutoff remains.

3.4. Discussion on the acceleration time

In Section 3.2 we have demonstrated that, in the case of a continuous acceleration in a medium which confines the particles, there is an energy or rigidity range where the spectrum presents a cutoff. Even if the source function is not limited in energy, as when $q(E, t)$ represents a thermal source, this cutoff still exists. In all cases, the presence of region 2 has the same effect on the observed spectrum as that of a sharp cutoff close to the lower edge of this region.

In these conditions, the experimental values obtained for R_m have to be examined in connection with the results of the preceding section.

The simplest hypothesis consists in assuming that all the parameters concerning the accelerating medium, and particularly α, remain the same for all the events. Thus, R_m is a monotonic function of the duration t_{ac} of the acceleration only. In these conditions, the knowledge of t_{ac} is sufficient to determine α and other parameters. Several authors (e.g. Ellison *et al.*, 1961; Švestka, 1970) propose that the acceleration of the particles occurs during the flash phase of the Hα flare. The duration Δt of this phase then possibly represents that of the acceleration. We do not find any clear relation between R_m and Δt. Indeed, in the case of the February 23, 1956 and the July 18, 1961 events, the upper cutoffs of which are respectively 20 GV and 4.3 GV, we obtain respectively $\Delta t = 8$ mn and $\Delta t = 18$ mn. In the same way, t_{ac} can be derived from the time profiles of the impulsive hard X rays bursts (e.g., Švestka, 1970) or from impulsive microwave bursts which have equivalent time profiles (e.g., Kundu, 1961). Here again we do not find any correlation between R_m and Δt. In fact the time profiles of the microwave bursts at 9400 MHz are quite similar, with different amplitudes, for the cases of the February 23, 1956 and January 24, 1971 events, the upper cutoffs of which are respectively 20 GV and 4.2 GV. In these conditions, any temporal parameter which can be determined from these two profiles, will be quite similar for the two events. Recently, Švestka and Fritzová-Švestková (1974) have shown that high energy particle events are closely connected with the occurrence of Type II bursts which, according to them, indicates that acceleration by shock waves occurs on the Sun. As we find that R_m remains constant for several hours, the shock front could accelerate particles along only a short distance. This is confirmed by the fact that, in some cases, the high energy particles hit the Earth a short time after the onset of the responsible flare. The lack of experimental indications on the path along which particles are accelerated prevents us from determining t_{ac} in this case.

The above discussion shows that (i) the simple hypothesis made is not valid; or (ii) the examined parameters are unable to give the duration of the acceleration, or (iii) the acceleration does not occur in a continuous way.

We now examine the case of α varying from one event to another. We have experimentally shown that the spectrum is represented by a power law. Moreover, the exponent of this law does not vary very much between all studied events, any slight variation being partly due to the propagation in the interplanetary medium. From expression (3-5) this implies a model in which T must vary from an event to another in the opposite way to α. Moreover, in this case, we have no criteria to determine t_{ac} from the observations. Indeed, even in the simple case $\alpha = bE$, keeping the others parameters fixed, a choice of t_{ac} is sufficient to calculate α for each event, but neither the values of α nor the choice of t_{ac} can be justified.

Finally the acceleration may not necessarily be represented by an equation of the type 3-1. An example of this can be found in the model proposed by Carlqvist (1969). According to this model E_m shows the effective potential which accelerates the particles to high energies.

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