On the Origin of the Upper Cutoff in the Solar Proton Spectrum

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The problem of the origin of the upper cutoff in the spectrum of multi-GeV solar protons is examined. This analysis concerns the case in which the upper cutoff is determined at the source level by a stochastic mechanism of acceleration. The operation of the random process at the solar source is justified in terms of a small length scale of the fluctuations of the magnetic field. The deviation of the particle spectrum from a perfect power law is explained by a discontinuity in the behavior of the mean free path of particles for acceleration. The assumption that this random acceleration step depends highly on the medium density (10^{13} cm^{-3}) and magnetic field strength (500 G) leads us to estimate a maximum rigidity greater than the measured value. We also found that the upper cutoff is imposed not by the linear dimension of the source but by this characteristic length scale of turbulence therein and that the source dimension is somewhat smaller than is usually thought. From the magnitude of some physical parameters of the source estimated in this paper the scale of turbulence of the magnetic field of about 1 km seems to be an interesting result within the framework of stochastic acceleration theory.

1. Introduction

The concept of the upper cutoff in the spectrum of cosmic ray particles accelerated within a source of finite volume has been previously mentioned in the literature as a steepening of the spectrum, starting at some energy value beyond which the particle-generating process is no longer governed by the same statistical parameters [e.g., *Morrison et al.*, 1954; *Burbidge and Hoyle*, 1964]. In the case of cosmic ray protons of very high energies ($>10^{15}$ eV) a break in the primary spectrum, leading to a high-energy cutoff, is explained by *Barrowes* [1971] on the basis of limitations imposed on the acceleration mechanism by cosmic ray absorption in supernova envelopes. Other interpretations are associated with energy loss processes, i.e., the energy degradation during their propagation following the interaction of particles with the microwave flux of the universal 3°K blackbody radiation [e.g., *Greisen*, 1966; *Zatsepin and Kuzmin*, 1966; *Hillas*, 1968].

In the case of solar electrons, *Svestka* [1970] has deduced a high-energy cutoff at about 2–18 MeV on the assumption that the synchrotron energy losses are greater than the acceleration rate and that a magnetic field strength of 10^3 G is present at the source. Nevertheless, he admits a higher value (14–87 MeV) if the field strength is $\sim$500 G. Following almost the same physical assumptions, *Sakurai* [1971] suggests a greater high-energy cutoff E_m = 5 GeV under the consideration that the magnetic field strength in the source region is only 10–100 G and that the acceleration process is performed by a Fermi mechanism. Previously, *Takakura* [1967] had analyzed an upper limit of electron acceleration, also on the basis of the Fermi mechanism. These evaluations of the higher cutoff are of course only theoretical. So far as we know, there is at present no systematic measure of the upper cutoff in the spectrum of solar electrons for individual flares, as exists in the case of solar protons. However, it may be mentioned that *Simnett* [1973] has analyzed solar electron spectra during five events in the period 1968–1969, finding a characteristic spectrum of the form $E^{-\gamma}$ with γ = 3 ± 0.4 below 10 MeV, which becomes steeper beyond 10 MeV ($\gamma \geq 3.5$). This result introduces the possibility of an upper cutoff at $\sim$100 MeV.

Here we are interested in investigating the nature of the observed upper cutoff in the high-energy portion of the spectrum of solar protons, the existence of which has been demonstrated by *Heristchi and Trottet* [1971] and *Heristchi et al.* [1972] for some proton events in the GeV range and supported statistically by the analysis of all proton events with detectable effects at ground level since 1956 [*Pérez-Peraza*, 1972; Dj. Heristchi, J. Pérez-Peraza, and G. Trottet, manuscript in preparation, 1975].

It is then adequate to emphasize that by high-energy cutoff we mean a discontinuity in the solar particle spectrum beyond a certain energy at which protons are no longer accelerated and consequently decrease in intensity more rapidly than a perfect power law predicts. For calculation purposes the upper cutoff has been taken as an abrupt break in the spectrum.

The results of the analysis performed by these authors show that the upper cutoff of solar protons in terms of magnetic rigidity ranges from 2.8 to 7.2 GV (e.g., 5.7 ± 1.5 GV and 3 ± 0.2 GV in the cases of the November 18, 1968, and September 1, 1971, events, respectively) and the exponent μ in the rigidity power law assumed for the differential spectrum has a range within the interval $1.6 < \mu < 5.7$ in different solar proton events. It should be noted that the method used to measure the upper cutoff prevents a clear distinction between the particle spectrum slope above and below the upper cutoff, because as has been remarked by *Heristchi and Trottet* [1971], the cutoff may be gradual and not so sharp as that assumed for calculation purposes. In fact, the spectrum slope at high energies is strongly dependent on the specific yield function for protons. Nevertheless, a rough idea of the change in the spectrum slope may be seen, for example, from the January 28, 1967, event illustrated in the work just mentioned, which shows a strong steepening of the spectrum above the cutoff, that is, a change from $\mu \sim 4$ to $\mu \sim 4.2$ approximately. As is shown by *Heristchi and Trotter* [1971], the time constancy of the high-energy cutoff during a solar cosmic ray event and the unperturbed behavior of this parameter during propagation of protons in the interplanetary medium lead to the conclusion that this upper cutoff is defined at the source level and not during particle propagation to the earth. Hence we believe that any explanation of the observed upper cutoff must be developed in terms of a similar effect in the source acceleration spectrum.

Concerning the energy gain process which gives rise to suprathermal particles in solar flares there are two principal ways of regarding it: either from the point of view of a stochastic irreversible process or that of a global reversible mechanism [*Schatzman,* 1970]. In section 2 we shall proceed to set out the theoretical problem, and in sections 3 and 4 we shall derive our assumption about the upper cutoff in the spectrum of solar protons from the first point of view on the basis of a Fermi-type mechanism, which is the kind of process actually evoked in order to explain several features of solar nuclei abundances (e.g., *Cartwright and Mogro-Campero* [1973], *Ramudarai* [1973], and several papers presented at the Thirteenth International Cosmic Ray Conference held in Denver, Colorado, in 1973).

2. Departure of the Solar Proton Spectrum From a Perfect Power Law

When a stochastic process is considered, as, for example, the acceleration of charged particles by isotropic gas magnetic turbulence fields (which are not in thermodynamic equilibrium), the behavior of particles at the source leads systematically to a consideration of the random properties of the interactions of particles with the electromagnetic field by means of a diffusion equation similar to a kinetic Fokker-Planck equation [*Tsytovich,* 1966]. It is well-known that in such cases the total number of accelerated particles $N(R, t)\, dR$ in the rigidity range between R and $R + dR$ as a function of time and rigidity can be determined by solving the following equation:

$$\frac{\partial N(R, t)}{\partial t} + \frac{\partial}{\partial R}\left[N(R, t)\frac{dR}{dt}(R, t)\right] + \frac{1}{\tau}N(R, t) = q(R, t) \qquad (1)$$

where τ is the mean confinement time in the acceleration region (which in general is expected to be a decreasing function of rigidity), $q(R, t)$ is the injection spectrum (injection rate per unit time furnished by the sources in a certain interval of rigidities), and dR/dt is the continuous rate of energy change (acceleration and deceleration) which is actually a function of rigidity and time. Thus in order to solve the continuity equation (1) we can make the following simplifications: we shall consider a stationary case under the assumption that the confinement time τ is short in comparison with the total duration of the acceleration process, in which case the steady state is promptly attained. For the injection rate one can assume a critical rigidity of injection R_i such that $q(R, t) = 0$ for $R > R_i$ or such that the injection rate is constant in time. In both cases we have a similar solution for the spectral shape of the accelerated particles. By taking, for example, the last option, (1) may be reduced to

$$\frac{\partial}{\partial R}\left[N(R)\frac{dR}{dt}(R)\right] + \frac{N(R)}{\tau} = q(R) \qquad (2)$$

When removal by nuclear absorption within the parameter τ is disregarded, (2) will only be influenced by leakage from the acceleration volume. Concerning the rate dR/dt in a first approximation we can neglect the energy losses during the acceleration, considering that acceleration occurs only when the energy gain rate overtakes the rate of energy losses; we then have for the steady state

$$dR/dt = \alpha(R)\cdot R \qquad (3)$$

where $\alpha(R)$ is the acceleration efficiency, which in general must be a function of rigidity and time. Of course, we are here considering the possibility that both α and τ depend only on rigidity. However, in order to obtain a power law, as was done in the observed spectrum of cosmic rays, it has often been assumed for the solution of the continuity equation (2) that both the confinement time τ and the acceleration efficiency α are independent of rigidity; that is, the mean free path for scattering with magnetic scatter centers is independent of rigidity and depends fundamentally on the spacing between the scatter centers. In other words the parameters α and τ are considered roughly as being constant. In addition, we shall also consider here the case for which α and τ depend on R but in such a way that $\alpha(R)\cdot\tau(R) \simeq$ const. In both cases the solution of (2) gives a differential spectrum of the form

$$N(R) \sim R^{-(1+1/\alpha\tau)} \qquad (4)$$

In fact, this spectral shape is independent of the acceleration mechanism (irreversible or reversible) provided that the acceleration rate satisfies (3). The stochastic picture described above relates to the Fermi statistical mechanism that will be discussed later in section 3.

From the examination of (4) it then appears that the analysis of two main situations, depending on the structure of the source, is of interest: first, in some way both α and τ vary gradually with increasing energy such that their product remains constant through the different rigidity bands in the course of the whole process, or second, α and τ remain constant up to some definite value of the rigidity, and then one or both of the two parameters suddenly begin to decrease in a more or less abrupt manner, in which case the intensity of accelerated particles rapidly drops off.

It is then clear that in order to maintain the spectrum (4) with a perfect power law shape the first alternative must be considered. Consequently, if the spectrum presents a certain kind of break in its high-rigidity portion, the second alternative must be considered. We shall therefore proceed to explain the observed upper cutoff in the solar proton spectrum in this last manner, developing some ideas which have been discussed earlier by *Hayakawa* [1969].

3. Justification of the Upper Cutoff in the Particle Spectrum

Concerning the generating process of solar cosmic rays it has been sometimes argued that a Fermi-type mechanism is not able to act in association with the acceleration of solar flare particles. This kind of argument follows from the slowness in the random feature of the process, which is inadequate for explaining the cosmic ray production within the time scale required by the flare phenomenon ($\leq$120 s), and also from the strong injection conditions which are necessary for the effective operation of this process in the high-density medium of the solar atmosphere (where different kinds of collisions with the traversed matter act as a decelerating process). Nevertheless, the most quantitative work at present on solar particle acceleration seems to show that a Fermi-type acceleration process is actually best able to explain the different features of the solar particle spectrum [e.g., *Parker,* 1957, 1958; *Wentzel,* 1964, 1965; *Fichtel and McDonald,* 1967]. Also the observations reported by *McLean et al.* [1971] strongly support this mechanism. Futhermore, we shall develop in the next sections some arguments supporting a high efficiency of the energy gain process and weak injection conditions when the acceleration region is characterized by a small-scale structure of the flare magnetic fields. In order to explain an upper cutoff in the acceleration spectrum on the basis of this mechanism we shall first assume that the source region has effectively a field pattern sufficiently irregular to allow us to give a simplified description of the acceleration process by means of the diffusion approximation. Given that in such a description it is supposed that the magnetic scatter centers have a wide distribution of sizes and field strengths, we shall develop our reasoning in terms of average quantities. Following the diffusion picture it is well-known that the random walk step of particles between magnetic scatter centers of average diameter d in the acceleration volume of linear dimension L is performed with a mean free path for scattering λ, which can be regarded as the mean distance within which a particle changes its original direction. Hence the mean free path before leaving the diffusion volume is $\Lambda \simeq n\lambda$, where $n = (L/\lambda)^2$ is the total number of collisions of particles with the diffusion centers. As the mean life of protons is mainly determined by the escape probability from the acceleration region, we have $\tau = \Lambda/v$, where v is the average particle velocity. Since in the small range of rigidities considered in the work of *Heristchi and Trottet* [1971] (some gigavolts) the increase of v is very slow, v can be taken as a constant and approximated to the light velocity c in such a form that we can make use of the well-known expression

$$\tau = L^2/\lambda c \tag{5}$$

Beginning with the examination of this parameter τ in (4), we know, according to the second hypothesis of the last section, that in order to maintain the confinement time τ roughly constant during acceleration the mean free path λ must be independent of the particle rigidity. Let us therefore discuss the conditions under which this situation is realized. In general, the dependence of λ on rigidity can be understood in terms of the Larmor radius r as

$$\lambda = r^2/l \tag{6}$$

where l is the average distance between scatter centers, r is the gyroradius (given in cgs units as $r = R \sin\theta/300H$), R is the magnetic rigidity of the particle, H is the magnetic field strength, and $\theta \simeq l/r$. However, depending on the distribution of the diffusion centers there are two extreme cases for which the mean free path λ may be taken as being constant. In the first case the scatter centers are widely spaced, and λ may be expressed in terms of the total number of collisions as $\lambda \simeq l/(n)^{1/2}$. It follows that in those cases, $l \simeq L$, and consequently $\lambda \simeq L = \text{const}$. In the second case we have the opposite situation (the so-called 'thick scattering' geometry); that is, the centers of diffusion are so numerous (i.e., when there is a high density of the fluctuations of the electromagnetic field) that in consequence their average distance l is very short. In this case the mean free path may be expressed in terms of the mean diameter d of the scatter centers, as $\lambda = l^2/d$ (see Appendix 1). It follows that in the situation for which $l \simeq d$ we have $\lambda = l = \text{const}$.

We believe therefore that in the solar case it is probably the latter condition which occurs, because a high-energy cutoff imposed by the source dimension L is insignificant, as was true in the first case. In fact, the high-energy cutoff should be in this last case several orders of magnitude above the observed values (as will be discussed in section 4).

We assume here that while the particle Larmor radius is inferior to the average distance between inhomogeneities of the magnetic field ($r < l$) (that is, for the case in which $R < 300Hl/\sin\theta$), we can approximate the magnitude of λ to the order of the mean distance l in such a way that (5) becomes $\tau \simeq L^2/lc$ and therefore independent of energy. On the other hand, if the rigidity of particles increases to such a degree that $R > 300Hl/\sin\theta$, when $r > l$, according then to (6), λ can no longer be considered as being of the same magnitude as l but becomes an increasing function of rigidity. In consequence, as the distance λ in which the scatter centers are effective increases, the collisions become less frequent because $n \sim \lambda^{-2}$, and obviously, the acceleration process is then less efficient. This result leads of course to a strong decline in the particle intensity at high energies, giving probably, as we have already mentioned, the observed break on the spectrum. This discontinuity on the particle spectrum can be seen from (4) by the fact that the mean confinement time changes from $\tau \simeq L^2/lc$ to $\tau' \simeq L^2l/r^2c$. However, the observed decline in intensity may not be very steep but rather of a gradual nature. Thus a description of the spectrum with a slope corresponding to τ' can be meaningless in practice, and therefore the spectrum slope can be approximately determined by τ.

We think that the discontinuity in the particle spectrum that we have just discussed is specifically in the case of solar protons the upper cutoff observed by *Heristchi and Trottet* [1971]. The high-rigidity cutoff of solar protons determined by the cessation of the accelerating process at $r = l$ is then in terms of magnetic rigidity

$$R_m = 300Hl \tag{7}$$

where H and l are the average values of the stochastic field strength and the random acceleration step, respectively (see Appendix 2).

Concerning the other parameter α we have just mentioned that the acceleration efficiency falls off strongly when λ increases with energy. This result can be illustrated for relativistic particles ($W = pc = zeR$), in which case this parameter is generally expressed for a stochastic acceleration mechanism as

$$\alpha \simeq \left(\frac{L}{\lambda}\right)^2 \frac{\Delta R}{R} \cdot \frac{1}{t_a} \tag{8}$$

with t_a being the characteristic acceleration time. It can be seen then from (4) that this variation of α with λ amplifies still more the break on the rigidity spectrum.

It is worth mentioning that even though the particle scattering behavior described here is a simpler diffusion picture, it is, however, in good agreement with the more rigorous models of particle scattering, for example, with the model of *Parker* [1964] of particle scattering by small-scale magnetic field irregularities in large-scale fields. In such a case, when $l/r < 1$ (which we assume to be typical in the solar source), the scattering function rapidly reaches zero. This can be seen, for example, with a scattering function of the form $(l/r - k) \exp(-l^2/2r^2)$ (with l and r corresponding to b and R, respectively, in Figure 2 of *Parker* [1964] and $k \simeq 0.5$).

4. Evaluation of the Actual Energy Cutoff

In order to estimate the magnitude of the high-energy (or rigidity) cutoff by means of (7) it is interesting to note that even though the expression was derived in the last section from the assumption of a stochastic acceleration process, *Syrovatskii* [1969] has shown (on the basis of the conservation of generalized momentum of particles along a neutral magnetic sheet) that such a relation is also operative in those cases in which reversible acceleration occurs. In the latter circumstances the parameter l does not of course necessarily retain the same meaning, but it can be identified, for instance, with a neutral sheet length.

For the evaluation of the order of magnitude of the magnetic field strength concerned it must be noted that the distribution of the magnetic field strength in the flare region is not actually well-established. Hence we base our findings on the following observations. *Howard* [1959], using spectroheliographic techniques, found that the magnetic field in active regions of the lower solar atmosphere is confined to very small areas where the field must be very strong. These measurements are consistent with the results reported by *Severny and Bumba* [1958], indicating a magnetic field strength in flares of the order of 300–500 G. In addition, the estimates of energy balance carried out by *Parker* [1957] in association with chromospheric flares indicate that at least in the case of the flare of February 23, 1956, a magnetic field of $\sim$500 G was present. Recently, the observations reported by *Newkirk* [1967] indicate that the magnitude of the magnetic flux density in an active region decreases from 1000 G at the photospheric level to 1 G at an altitude of about 1 R_S above the photosphere according to a law of type $\sim S^{-10}$ (S being distance above the photosphere). On the other hand, we know from the magnetographic measures of *Howard* [1962] that an rms magnetic field of $\sim$10 G characterizes the quiet regions of the solar atmosphere. Hence taking into account the stochastic character of the magnetic field fluctuations, we shall assume that magnetic inhomogeneities in active regions have on the average a strength as high as 500 G, while the background field is weaker by one or more orders of magnitude. In fact a magnetic field of 500 G in flare structures seems to be at present an accurate estimate [e.g., *Svestka,* 1970; *Krivsky,* 1970].

The magnitude of the parameter $l \simeq \lambda$ is actually one of the main uncertainties connected with the statistical mechanism; nevertheless, an approximate evaluation may be obtained by following the diffusion picture carried out by *Parker* [1957], which allows us to represent l as a function of the medium concentration N and the magnetic field strength H (see Appendix 3):

$$l(N, H) = 1.42 \times 10^{34} H^4/[N^3(1.5 \ln H + 29.0 - \ln N)] \quad (9)$$

We have tabulated in Table 1 the results obtained from (7) and (9) for the range of particle concentrations existing in solar flares, 10^9–10^{14} cm^{-3}, and some values of the magnetic field strength between 50 and 1000 G. We have considered that for a field strength weaker than 50 G we are dealing, as was mentioned above, with a nonperturbed region (the local magnetic field), and for a field strength stronger than 1000 G we are, according to *Newkirk* [1967], outside 1 R_S.

From the examination of Table 1 it can be seen that there are several pairs of values (N, H) furnishing upper cutoff rigidities of the order of magnitude of the observed average value $R_m \simeq 5$ GV. This is deduced from the analysis of all multi-GeV proton events of the last two decades [*Pérez-Peraza,* 1972]. Therefore the acceleration region could be found in a region where the particle concentration is of the order of 10^{11}–10^{13} cm^{-3}. However, it must be considered that at particle densities of $\geq 10^{11}$ cm^{-3}, inhibition in the acceleration efficiency owing to losses of energy must be taken into account [e.g., *Kaplan,* 1956; *Parker,* 1957; *Morrison,* 1961; *Ramudarai,* 1971]; that is, physically, it is difficult to conceive that charged particles could traverse a dense medium without losing some fraction of their energy during and after their acceleration. To include the effects of energy losses after acceleration, we must choose a pair (N, H) furnishing a rigidity greater than the observed upper cutoff, but also in these circumstances, owing to the strong sensibility of (9) to the parameters N and H, it is possible, as is shown in Table 1, to find various pairs (N, H) satisfying this condition in the density range explored. Following this reasoning one can conclude in a first approximation that the acceleration region is found in a region where N is between 10^{11} and 10^{13} cm^{-3} and that the length scale for inhomogeneities of the magnetic field is of the order of 10^5–10^6 cm. It follows that in order to place the source region in the upper atmosphere, where $N < 10^{11}$ cm^{-3}, a magnetic field strength lower than the values tabulated in Table 1 should be assumed. However, as we have already pointed out, magnetometric measures seem to indicate that the fields as-

TABLE 1. High-Rigidity Cutoff and Random Acceleration Step of Protons for Some Typical Values of the Particle Concentration and the Magnetic Field Strength in the Solar Atmosphere

N, parts/cm^3	H = 1000 G		H = 500 G		H = 300 G		H = 100 G		H = 50 G	
	l, cm	R_m, V	l, cm	R_m, V	l, cm	R_m, V	l, cm	R_m, V	l, cm	R_m, V
10^9	7.6×10^{17}	2.3×10^{23}	5.0×10^{16}	7.6×10^{21}	7.0×10^{15}	6.1×10^{20}	9.3×10^{13}	2.8×10^{18}	6.2×10^{12}	9.4×10^{16}
10^{10}	8.7×10^{14}	2.6×10^{20}	5.8×10^{16}	8.7×10^{18}	8.0×10^{12}	7.1×10^{17}	1.1×10^{11}	3.3×10^{15}	7.5×10^9	1.1×10^{14}
10^{11}	1.0×10^{12}	3.0×10^{17}	6.8×10^{10}	1.0×10^{16}	9.4×10^9	8.5×10^{14}	1.3×10^8	4.0×10^{12}	9.3×10^6	1.4×10^{11}
2×10^{11}	1.3×10^{11}	4.0×10^{16}	9.0×10^9	1.3×10^{15}	1.2×10^9	1.1×10^{14}	1.8×10^7	5.4×10^{11}	(1.2×10^6)	(1.8×10^{10})
3×10^{11}	4.0×10^{10}	1.2×10^{16}	2.8×10^9	4.1×10^{14}	3.8×10^8	3.5×10^{13}	5.5×10^6	1.7×10^{11}	(3.9×10^5)	(5.8×10^9)
4×10^{11}	1.8×10^{10}	5.3×10^{15}	1.2×10^9	1.8×10^{14}	1.6×10^8	1.5×10^{13}	2.4×10^6	7.2×10^{10}	1.7×10^5	2.6×10^9
5×10^{11}	9.0×10^9	2.8×10^{15}	6.2×10^8	9.3×10^{13}	8.6×10^7	7.8×10^{12}	(1.2×10^6)	(3.8×10^{10})	8.9×10^4	1.3×10^9
6×10^{11}	5.4×10^9	1.6×10^{15}	3.7×10^8	5.5×10^{13}	5.1×10^7	4.6×10^{12}	(7.5×10^5)	(2.2×10^{10})	5.3×10^4	7.9×10^8
7×10^{11}	3.4×10^9	1.0×10^{15}	2.3×10^8	3.5×10^{13}	3.3×10^7	3.0×10^{12}	(4.8×10^5)	(1.4×10^{10})	3.4×10^4	5.1×10^8
8×10^{11}	2.3×10^9	7.0×10^{14}	1.6×10^8	2.4×10^{13}	2.2×10^7	2.0×10^{12}	(3.2×10^5)	(9.8×10^9)	2.3×10^4	3.4×10^8
9×10^{11}	1.6×10^9	5.0×10^{14}	1.1×10^8	1.7×10^{13}	1.5×10^7	1.4×10^{12}	(2.3×10^5)	(7.0×10^9)	1.6×10^4	2.4×10^8
10^{12}	1.2×10^9	3.6×10^{14}	8.3×10^7	1.2×10^{13}	1.1×10^7	1.0×10^{12}	(1.7×10^5)	(5.0×10^9)	1.2×10^4	1.8×10^8
2×10^{12}	1.6×10^8	4.8×10^{13}	1.1×10^7	1.7×10^{12}	1.5×10^6	1.4×10^{11}	2.3×10^4	7.0×10^8	1.7×10^3	2.5×10^7
3×10^{12}	5.0×10^7	1.5×10^{13}	3.4×10^6	5.1×10^{11}	(4.8×10^5)	(4.3×10^{10})	7.3×10^3	2.2×10^8	5.3×10^2	8.0×10^6
4×10^{12}	2.1×10^7	6.4×10^{12}	1.5×10^6	2.2×10^{11}	(2.1×10^5)	(1.9×10^{10})	3.2×10^3	9.7×10^7	2.3×10^2	3.5×10^6
5×10^{12}	1.1×10^7	3.3×10^{12}	7.8×10^5	1.2×10^{11}	(1.1×10^5)	(9.9×10^9)	1.7×10^3	5.1×10^7	1.2×10^2	1.8×10^6
6×10^{12}	6.6×10^6	2.0×10^{12}	4.6×10^5	7.0×10^{10}	(6.5×10^4)	(6.0×10^9)	1.0×10^3	3.0×10^7	75.0	1.1×10^6
7×10^{12}	4.2×10^6	1.3×10^{12}	3.0×10^5	4.4×10^{10}	4.2×10^4	3.8×10^9	6.0×10^2	2.0×10^7	49.0	7.3×10^5
8×10^{12}	2.9×10^6	8.6×10^{11}	(2.0×10^5)	(3.0×10^{10})	2.8×10^4	2.5×10^9	4.4×10^2	1.3×10^7	33.0	5.0×10^5
9×10^{12}	2.0×10^6	6.1×10^{11}	(1.4×10^5)	(2.1×10^{10})	2.0×10^4	1.8×10^9	3.2×10^2	9.6×10^6	24.0	3.6×10^5
10^{13}	1.5×10^6	4.5×10^{11}	(1.0×10^5)	(1.6×10^{10})	1.5×10^4	1.3×10^9	2.3×10^2	7.1×10^6	18.0	2.7×10^5
2×10^{13}	(2.0×10^5)	(6.0×10^{10})	(1.4×10^4)	2.2×10^9	2.0×10^3	1.8×10^8	33.0	1.0×10^6	2.6	3.9×10^4
5×10^{13}	(1.4×10^4)	(4.4×10^9)	1.0×10^3	1.6×10^8	1.5×10^2	1.3×10^7	2.6	7.8×10^4	...	3.2×10^3
7×10^{13}	5.5×10^3	1.6×10^9	4.0×10^2	6.0×10^7	60.0	5.3×10^6	1.0	3.0×10^4	...	1.3×10^3
10^{14}	2.0×10^3	6.0×10^8	1.5×10^2	2.2×10^7	21.0	2.0×10^6	...	1.1×10^4	...	5.0×10^2

Values in parentheses correspond to the more plausible values of these parameters in the sense that they are high and nearest the measured values of R_m.

sociated with solar flares are of the order of several hundred gauss.

Hence it is clear that if we want to go deeply into the determination of the source region by this reasoning, the magnetic field strength must be defined; our assumption of a field strength of about 500 G results in obtaining from Table 1 a theoretical value of $R_m \simeq 16$ GV. This has been chosen to be high (in order to take into account energy losses) but at the same time to be the nearest value to the measured one. It may be seen that this value corresponds to a length scale of the magnetic field $l \simeq \lambda \simeq 1$ km and a medium density $N \simeq 10^{13}$ cm^{-3}. It must be pointed out that even though this value of R_m is several times higher than the measured average value, it is of the same order as that of the exceptional event of February 23, 1956, for which there is evidence of cosmic ray production with rigidities far above the average of the measured values (~5 GV).

We would now emphasize that a high-energy cutoff imposed by the linear dimension of the source L is irrelevant in the case of solar proton generation, as has been previously argued; in view of this conclusion we remark that concerning the linear dimension L of the solar proton source there is a certain diversity of opinion about its magnitude, varying from $\sim 10^8$ to 10^{11} cm. Nevertheless, we shall proceed to discuss later some arguments supporting a dimension somewhat smaller. In any case these values would furnish an upper cutoff rigidity $R_m \simeq 300HL \simeq 10^{13}$–$10^{17}$ V in a region where the magnetic field strength is of the order of 500–1000 G [*Svestka*, 1970; *Krivsky*, 1970]. These values of the upper cutoff would obviously be very far above the observed value. This finding implies then that at least in the case of the solar source the maximum rigidity transferred to protons depends not on the linear dimensions L of the region, as was usually assumed in cosmic ray physics [e.g., *Cocconi*, 1960], but on the random acceleration step $\lambda \simeq l$.

A scheme of the physical structure of the source in the flare region may now be deduced from the framework of the Fermi statistical mechanism, which is a typical example of stochastic processes: in this case then under the assumption of a characteristic length scale $l \simeq \lambda \simeq 1$ km for the fluctuations of the magnetic field the acceleration efficiency is $\alpha_f = V^2/cl \simeq 0.4$, when $V = H/(4\pi\rho)^{1/2} \simeq 3.5 \times 10^7$ cm/s is the hydromagnetic velocity in a medium of $N \simeq 10^{13}$ cm^{-3} and $H \simeq 500$ G. Concerning the mean confinement time τ, which is also a nonmeasurable parameter, there are several indirect ways of evaluating it. This evaluation can be made from the point of view of a diffusion process that is either energy-dependent [*Webber*, 1964] or velocity-dependent [*Bryant et al.*, 1965]. In the latter case, *Wentzel* [1965] has deduced $\tau \simeq 3$ s for protons and alpha particles above 2 MeV. On the other hand, it is also possible to estimate it from the comparison of the observed spectrum slope with the theoretical slope (equation (4)), that is, $\gamma = 1 + 1/\alpha\tau \sim 2$–6, or $\tau \simeq 0.5$–2.5 s. One must therefore expect that $L = (c\lambda\tau)^{1/2} \simeq (3.9\text{–}8.7) \times 10^7$ cm, which is of course smaller than the linear dimensions usually attributed to the solar proton source.

It follows then that under the assumption of such a source structure the critical kinetic energy of injection for protons $E_i = 3.2 \times 10^{-7} N(lc/V^2)\,[1.02 - (1/84) \ln N]$ [*Dorman*, 1960] is only ~2.5 MeV. In consequence, the acceleration up to ~15.1 GeV is performed in a time $t_a = (1/\alpha) \ln (E_m/E_i) \sim 22$ s, which is obviously within the time scale of the flare phenomenon. A summary of the physical parameters characterizing the solar source of proton acceleration is given in Table 2.

TABLE 2. Physical Parameters Characterizing Solar Proton Sources

Parameter	Value
Number density, part/cm^3	$N \simeq 10^{11}$–10^{13}
Magnetic field strength, G	$H \simeq 500$
Random acceleration step scale of the magnetic field turbulence, cm	$\lambda \simeq l \simeq 10^5$–$10^6$
Velocity of magnetic irregularities, cm/s	$V_m \simeq 3.5 \times 10^7$
Linear source dimension, cm	$4 \times 10^7 \leq L \leq 9 \times 10^7$
Mean escape time, s	$0.5 \leq \tau \leq 3$
Acceleration time, s	$t_a \simeq 2$
Critical injection energy, eV	$E_i \simeq 2.5 \times 10^6$
Theoretical high-energy cutoff, eV	$E_m \simeq 15.1 \times 10^9$

5. SUMMARY AND DISCUSSION

The evidence given in favor of an upper cutoff in the solar proton spectrum by *Heristchi and Trottet* [1971] and the measures performed by *Pérez-Peraza* [1972] to determine the high-energy cutoff are undoubtedly a powerful tool for understanding the generation process of solar particles, which is not well-defined at present. The interpretation given to these measured values may or may not be adequate. However, the fundamental fact is that these are experimental parameters which must be taken into account, whatever the theoretical approach to the problem of the production of relativistic particles by the sun.

In section 2 we have analyzed the meaning of such an upper cutoff as a condition defined at the source level; to interpret this we have discussed in sections 3 and 4 the situation in which the particle spectrum is determined by the acceleration process, taking as an example a second-order Fermi mechanism. Nevertheless, we must point out that we do not disregard the possibility that the source spectrum of multi-GeV proton flares does not depend on the acceleration mechanism but is only determined from general thermodynamical requirements of discrete sources of cosmic rays (equipartition between the local density energy and the density energy of the solar energetic particles). In this case, as has been discussed by *Syrovatskii* [1961], the spectrum of particles leaving the source may be expressed as a power law in kinetic energy of the form $N(E)\,dE = \text{const}\; E^{-\gamma}\,dE$, with $\gamma = (2\kappa - 1)(\kappa - 1)$. The cosmic ray gas being regarded as an ideal gas, κ is a specific heat ratio varying from $\kappa = 5/3$ for nonrelativistic particles to $\kappa = 4/3$ for relativistic particles. Such a change could explain the observed steepening of the spectrum up to a certain maximun value of the energy reached when the source cavity can no longer retain the accelerated protons. This acceleration may perhaps be brought about by a global mechanism, as, for instance, the 'current disruption' mechanism suggested by *Alfvén and Carlqvist* [1967] and *Carlqvist* [1969].

In section 4 we have estimated the density of the generation region of the solar multi-GeV protons from the knowledge of the higher cutoff R_m, in which case the determination of the parameter H therein is necessary and the consideration of a value R_m just slightly higher than the measured value (to take into account energy losses) is sufficient. The consideration of any value higher than the measured value is insufficient because the cooling time necessary to reach this experimental value would be longer than the duration time of particles at the source, which is estimated as <120 s [e.g., *Webber*, 1964; *Wentzel*, 1965]; in fact, it can be seen by a simple calculation that protons of 15–30 GV in a region of $N \sim 10^{12}$ cm^{-3} would be cooled to 5 GV by *p-p* nuclear collisions and ionization losses in an average time of ~ 135 s. Thus in addition to the acceleration time, protons would remain within the source region for a period longer than the one estimated by *Webber* [1964] on the basis of statistical considerations. Thus we have concluded that the generation region of multi-GeV protons is found at $N \simeq 10^{13}$ cm^{-3}. This estimate of a high-density medium seems to be supported by some experiments indicating that the composition of solar cosmic nuclei differs from that of the coronal matter but approaches more nearly (although it is not equal to) that of the upper photosphere or the chromosphere [e.g., *Anglin et al.*, 1973*a*, *b*]. A similar number density N in the acceleration region of solar electrons has been deduced by *Korchak* [1973]. Spectroscopic analysis of the flare region indicates also $N \simeq 10^{13}$ cm^{-3} [*Svestka*, 1966].

We argue then that the cooling of protons after their acceleration to the observed value of the higher rigidity cutoff may be realized in a fast energy degradation step at the source or in its vicinity. That is, we know that a charged particle moving in a medium which is not the vacuum can lose energy by ionization, excitation, energy interchange with free electrons, or nuclear collision. On the other hand, from the fact that the flare phenomenon seems to be associated with the expansion of a certain structure in the form of a magnetic bottle [e.g., *Sakurai*, 1965; *Schatzman*, 1967] it follows that particles can suffer adiabatic deceleration or also lose energy by viscosity and Joule dissipation in that cavity [*Syrovatskii*, 1961]. In fact, *Carlqvist* [1969] has already proposed an energy transfer of particles to the solar plasma and the surrounding magnetic field. A model of proton generation with an energy degradation step together with its implications is being examined and will be discussed elsewhere. We may argue that in this case there must be some flares in which particles are not subject to the degradation step in energy. This was probably the case in the event of February 23, 1956, for which there is evidence of proton production far above the average measured value.

We have mentioned in section 3 that the steepening of the spectrum must be very gradual, which is quite a reasonable assumption, since it is expected that the scale length of magnetic irregularities determining the random acceleration step has a very broad distribution with a maximum centered around $l \simeq 1$ km. However, in order to state the theoretical exponents in the power law dependence above and below the upper cutoff we can proceed to compare our results with the behavior of the exponents in the experimental power law spectrum of the January 28, 1967, event (already discussed in section 1). Assuming the parameters to be those shown in Table 2, with $\tau = 0.8$ s, for example (i.e., $L = 5 \times 10^7$ cm), we have from (4), $\mu = (1 + 1/\alpha\tau) \simeq 4$. Thus it can readily be seen that if we consider in (6) a particle gyroradius just a little larger than l (say, $r = 1.1l$), we immediately obtain a change of slope from $\mu \simeq 4$ to $\mu \simeq 4.3$, giving the abrupt cutoff observed in the experimental spectrum.

At this point we would like to emphasize the importance of the parameter l in the context of this work; in fact, we have assumed that acceleration practically ceases when the gyroradius of protons approaches the diameter of the fluctuations of the magnetic field ($r \simeq d$) and that the density of magnetic field fluctuations is so high that the situation $l \simeq d$ must be considered in the case of the solar source. This finding allows us to conjecture that under the assumption of a turbulence length scale $l \simeq 1$ km, even a second-order Fermi mechanism may become of interest in solar flares, since the acceleration efficiency will be sufficiently high ($\alpha \simeq 4 \times 10^{-1}$ s^{-1}). This implies then that the injection conditions are relatively low for the operation of this mechanism in solar active centers. Finally, we remark that the hypothetical ideas developed here could take on some concrete meaning if by means of the modern high-resolution techniques for solar magnetic field measurements there could be detected a certain characteristic magnetic length of the order of $\sim$1–10 km. In fact, it has been several times proposed that the Balmer line emission proceeds from a very small structure of 1–10 km in diameter [e.g., *Suemoto and Hiei*, 1959; *Hirayama*, 1961; *De Feiter*, 1964; *Svestka*, 1965].

APPENDIX 1

In order to estimate the mean free path of particles in a diffusion volume where the scatter centers have an average size d and are separated by an average distance l we shall proceed in the following manner.

Let us suppose that l is defined in such a way that there exists, on an average, one diffusion center at distance l by radian considered from a certain point of origin. Now, the probability of a particle, emitted in a random manner into an angle of 1 rad, colliding at the distance l with a diffusion center is given when the emission is within the angle d/l, as $P(l) = d/l$ (favorable case). In the opposite case we have of course a probability given by $(l - d)/l$. The probability of finding a diffusion center again after a new free path l is performed is

$$P(2l) = \frac{d}{l}\frac{l-d}{l}$$

and in the same manner

$$P(3l) = \frac{d}{l}\left(\frac{l-d}{l}\right)^2$$

and so on. We can then rewrite

$$P(nl) = \frac{d}{l}\left(\frac{l-d}{l}\right)^{n-1}$$

The mean free path λ may therefore be expressed as

$$\lambda = \sum_{n=1}^{\infty} nlP(nl) \Big/ \sum_{n=1}^{\infty} P(nl) \tag{10}$$

$P(nl)$ being a probability, the sum $\sum_{n=1}{}^{\infty} P(nl)$ must be the unit; that is, since $P(nl)$ is a geometric progression with d/l as first term and $(l - d)/l$ as general term, the sum is then given simply as

$$\frac{d}{l}\Big/\frac{1-(l-d)}{l} = \frac{d}{l-l+d} = 1 \tag{11}$$

On the other hand, performing the integration of $nlP(nl)$ with respect to the general term $(z - d)/z$, we obtain

$$nl\left(\frac{d}{l}\right)\cdot\frac{1}{n}\left(\frac{l-d}{l}\right)^n = d\left(\frac{l-d}{l}\right)^n = d\cdot p^n \tag{12}$$

(with p being the general term and $d \cdot p$ being the first term of the integrated series). The sum of this integrated series is then given by $dp/1 - p$. Taking the derivate with respect to p, one obtains

$$\frac{d(1-P)+d\cdot P}{(1-P)^2} = \frac{d}{[1-(l-d)/l]^2} = \frac{d}{(d/l)^2}$$

Equation (10) rests therefore as $\lambda = l^2/d$.

APPENDIX 2

Syrovatskii [1969] has shown that the upper limit of achieved energy of the accelerated particles can be found from

Hamilton equations for the generalized particle momentum in a given direction. If diffusion of magnetic field lines in the magnetic field evolution equation of the magnetohydrodynamics is neglected, the condition of freezing-in is valid, and then from the conservation of the generalized momentum of particles it follows that

$$d/dt\ [p + (ze/c)\ A] = 0 \qquad (13)$$

where p is the particle momentum, ze is the charge, A is the potential of the field, characterized by a constant value at each line of force, and $d/dt = \partial/\partial t + v \cdot \nabla$ is the convective derivate on the particle trajectory. If at some point of the particle trajectory a reinforcement in strength of the lines of force takes place (inhomogeneities of the magnetic field), the potential changes by the value ΔA, the particle reaching a momentum

$$p_{\max} = |p_0 - (ze/c)\Delta A| \simeq (ze/c)|\Delta A| \qquad (14)$$

It follows that the maximum magnetic rigidity in cgs units is

$$R_{\max} = p_{\max}c/ze = \Delta A \simeq 300Hl \qquad (15)$$

where $A \simeq Hl$ is the maximum magnetic flux per unit of length in the acceleration region.

APPENDIX 3

Parker [1957] has investigated the problem of the relaxation time in electrostatic interactions by analogy with the gravitational analysis performed by *Chandrasekhar* [1942]. In the electrostatic analogy an estimate was made of the mean path traversed by a charged particle before it was slowed down to the thermal velocity by interactions with the ions (of mass M and thermal velocity u_p) and electrons (of mass m and thermal velocity $u_e = u_p(M/m)^{1/2}$) of the plasma through which it moves. Parker finds that the effective free path for a proton of velocity v before being slowed down is

$$h(v) \simeq (3M^2v^6)\{32\pi Ne^4u_p{}^2 \ln [M^{3/2}v^2u_p/4(3\pi v)^{1/2}e^3]\}^{-1} \qquad (16)$$

where N is the number of hydrogen ions per unit of volume in the plasma and e is the electron charge. Since it is expected that protons have an initial thermal velocity $v \simeq u_p$ and fluid velocities in the acceleration region are of the order of the characteristic hydromagnetic velocity V, i.e.,

$$u_p \simeq v \simeq V = H/(4\pi NM)^{1/2} \qquad (17)$$

(16) can be reduced to

$$h(N, H) = (3H^4)\{512\pi^3N^3e^4 \ln [H^3/32\pi^2(3)^{1/2}N^2e^3]\}^{-1} \qquad (18)$$

Now, in order that there should be effective Fermi acceleration of nonrelativistic protons the energy gain rate ($\alpha = 4V^2/c\lambda$) must be higher than the rate of energy loss by electrostatic interactions ($b = -v/2h$) [*Fermi*, 1954] in such a way that using (17), we can write $V/\lambda > v/8h$, or $\lambda < 8h$. Hence we must have $\lambda < \lambda_{\max} = l = 8h(N, H)$, and therefore from (18)

$$l(N, H) = 1.42 \times 10^{34}H^4/\{N^3(1.5 \ln H + 29.0 - \ln N)\} \qquad (19)$$

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